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A Tensor-Product Parity Code for Magnetic Recording

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I. Introduction

An error-correcting code is usually employed in magnetic recording to ensure data reliability. Reed-Solomon (RS) codes are, by far, the most commonly used for this purpose. Since the RS code is a symbol-based code, it is suitable to correct burst, but not random, errors. To help in correcting random errors, the RS code is often concatenated with an inner parity code.

To limit the rate penalty, practical applications often use high rate parity codes with a block length on the order of 30 to 100 bits. For such a large block length, there is a high probability that the code will miscorrect, and hence propagate, errors. It is also likely that multiple error events will occur within one block and therefore go undetected by the code.

In order to overcome these deficiencies, we propose the use of a "tensor-product parity code" whose parity-check matrix is the tensor-product of the parity-check matrices of a short parity code and a BCH code. This code achieves the same performance as the constituent parity code, but with higher rate.

II. Tensor-Product Parity Code System

Let C_1 be a single parity code with block length k_1+1 and let C_2 be an (n_2, k_2) binary linear code. Let $H_2 = [h_{i,j}]$ be a parity-check matrix of C_2. We define C to be the code with the following parity-check matrix:

$$H = \begin{bmatrix} h_{1,1}K & h_{1,2}K & \cdots & h_{1,n_2}K \\ h_{2,1}K & h_{2,2}K & \cdots & h_{2,n_2}K \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_2-k_2,1}K & h_{n_2-k_2,2}K & \cdots & h_{n_2-k_2,n_2}K \end{bmatrix},$$

where K = [1 1 … 1] is a row vector of size k_1. The code C has the following property. Suppose we divide a code word of C into disjoint blocks of length k_1, and compute the parity bits for each block using C_1. The parity bits of all blocks must form a valid code word of C_2.

The code C is a special case of a tensor-product code [1]. We therefore refer to C as a *tensor-product parity code*. This code can be easily generalized to the case where C_1 is a multiple parity code. The message length k_1 of C_1 is called the *block size* of C.

The tensor-product parity code can be concatenated with an RS code C_3, but we will, instead, combine the codes by taking their common subspace. In a slight abuse of terminology, we will call C_2 the *inner code* and C_3 the *outer code*.

An optimal sequence decoder for a parity-coded system is a Viterbi detector that combines channel states and code states [2]. The decoder ensures that the states at the parity block boundary satisfy the parity constraint. Since the correct parity is not known in advance, decoding of the tensor-product parity-coded system begins with detection of the recorded bits using the Viterbi algorithm matched to the channel only. Then, the detected word is divided into disjoint blocks of length k_1, and the parity of each block is computed to form a word w. From the property of C, the word w must be a code word of C_2. We decode w using C_2 and the corrected parity bits are provided to a Viterbi detector reflecting channel states and parity code states. Finally, the resulting sequence is decoded by the outer decoder.

III. Performance Comparison

Consider the (468,410) RS code over GF(1024). This code has 580-bit redundancy and can correct 29 symbol errors. To construct a tensor-product parity code system with a similar code rate, we choose the codes C_1, C_2, C_3 as follows. Fix C_1 to be the single parity code with $k_1 = 10$. Let C_3 be a (468, k_3) RS code and choose C_2 to be a (468, k_2) binary BCH code such that the total redundancy $10(468-k_3) + 468 - k_2$ is close to 580 bits. To find optimal values of k_2 and k_3, we simulate the tensor-product parity code system using the Lorentzian model with normalized pulse width PW50 = 3.0 and additive white Gaussian noise. The equalizer is a 12-tap finite-impulse-response filter with a length-5 partial response target. The sector error rate (SER) is estimated by an analytical method based on a block multinomial model [3]. At the signal-to-noise ratio (SNR) of 20 dB, optimal parameters are $k_2 = 225$ and $k_3 = 434$, which gives SER = 10^{-10}.

Next we vary the block size k_1 and find optimal code parameters using the same technique. The SER as a function of block size is plotted in Fig. 1. For comparison, we also plot the SER of the concatenation of a single parity code and an RS code, whose total redundancy is approximately 580 bits. For the concatenation system, a high-rate parity code is preferred since a low-rate parity code only allows a weak RS code. In contrast, the tensor-product parity code system gives lower SER when it has a smaller block size. When the block size is large, the SER is comparable to the concatenation system since the best inner code C_2 is the trivial code with one code word.

Finally, we select the best parameters for the concatenation system and the tensor-product parity code system and compare their performance to that of the (468,410) RS code. The SER of the three systems are shown in Fig. 2. We see that the tensor-product parity system has a gain of approximately 0.3 dB over the RS code and almost 0.2 dB over the concatenation system at SER = 10^{-10}.

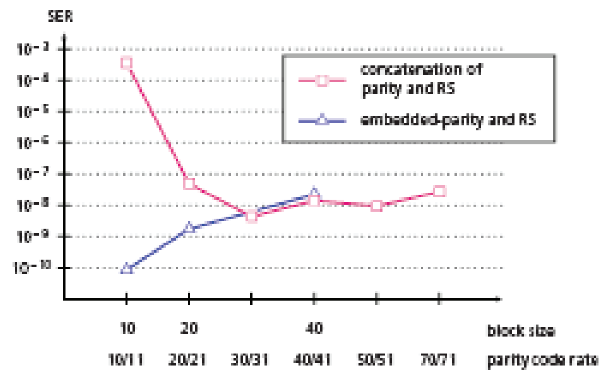

Fig. 1. Sector error rates at SNR = 20 dB of the tensor-product parity code system and the concatenation of a parity code with an RS code. The block size of the tensor-product parity code is varied from 10 to 40. For the concatenation system, the rate of the parity code is varied from 10/11 to 70/71.

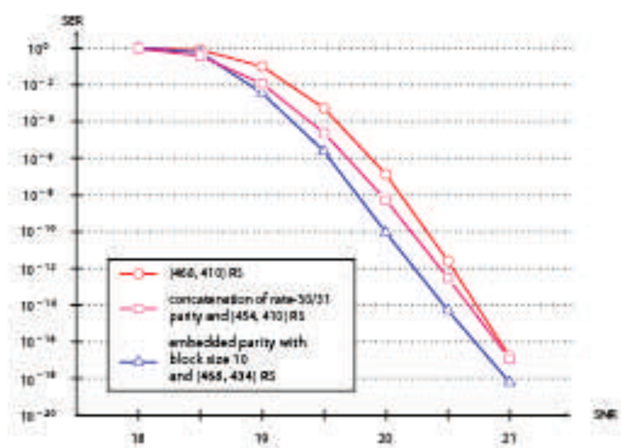

Fig. 2. Sector error rates of the (468,410) RS code, the concatenation of the rate-30/31 parity code and the (454,410) RS code, and the tensor-product parity code system with block size 10, (468,225) inner BCH code, and (468,434) outer RS code.

1. J. K. Wolf, “On codes derivable from the tensor product of check matrices,” *IEEE Trans. Inform. Theory*, vol. IT-11, no. 2, pp. 281-284, Apr. 1965.

2. Z. Wu, P. A. McEwen, K. K. Fitzpatrick, and J. M. Cioffi, “Interleaved parity check codes and reduced complexity detection,” in *Proc. ICC 1999*, Vancouver, BC, pp. 1648-1652.

3. Z. A. Keirn, V. Y. Krachkovsky, E. F. Haratsch, and H. Burger, “Use of redundant bits for magnetic recording: single-parity codes and Reed-Solomon error-correcting code,” *IEEE Trans. Magn.,* vol. 40, no. 1, pp. 225-230, Jan. 2004.