Channel Models for Multi-Level Cell Flash Memories Based on Empirical Error Analysis

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I. INTRODUCTION

Channel modeling for NAND flash memories is a developing research area with applications to better signal processing and coding techniques for flash memories. In [1], [2], the empirical cell threshold voltage distributions of multi-level cell (MLC) flash memories are modeled using known probability distributions to derive parametric channel models. The evaluation of such models is typically based on how well they are able to estimate/fit the empirically observed raw bit error rate (RBER) of the flash memory.

Analytical estimation of error-correcting code (ECC) frame error rate (FER) performance based on the RBER of a flash memory is hard especially for ECCs such as LDPC and Polar codes. Estimating the ECC FER performance for a flash memory experimentally using Monte-Carlo simulation techniques is also practically impossible due to the extremely large amount of empirical data required to estimate very low frame error rates (e.g., 10^{-10}). Hence it is important to develop parametric channel models for flash memories that can provide accurate ECC FER performance estimates.

In this abstract, based on a detailed empirical error analysis of 2Y-nm MLC flash memory chips we observe and characterize the *overdispersion* phenomenon in the *number of bit errors per ECC frame*. Due to this overdispersion phenomenon, we show that the RBER of a flash memory chip is not a sufficiently good predictor of the ECC FER performance and hence a well studied binary discrete memoryless channel model such as the binary asymmetric channel (BAC) model is unable to provide accurate ECC FER performance estimation. Hence we propose a binary discrete channel model based on the beta-binomial probability distribution (2-BBM channel model) and show using simulation results for BCH and LDPC codes, that the 2-BBM channel model provides accurate ECC FER performance estimation in MLC flash memories.

II. EXPERIMENT PROCEDURE

For error characterization, we perform program/erase (P/E) cycling of the MLC flash memory chip under test which consists of repeated application of the erase and program operations on the blocks under test. We also perform read operations at intervals of every 100 P/E cycles to record the bit error information. We arbitrarily select 4 contiguous blocks for our experiments and pseudo-random data is used for programming.

The blocks are P/E cycled up to 10,000 P/E cycles at room temperature in a continuous manner with no extra wait time between the erase/program/read operations.

III. ERROR CHARACTERIZATION

In this section, we study the empirical bit error characteristics during P/E cycling in MLC flash memories. We represent the four charge levels in MLC flash memory as 0, 1, 2, 3 in the increasing order of charge levels respectively. The corresponding 2-bit patterns written to the lower and upper pages are '11', '10', '00' and '01' respectively. The two main characteristics studied are the asymmetry of bit errors and the distribution of number of bit errors per frame.

TABLE I

ERROR CHARACTERIZATION RESULTS

(A)

Write Cell Values	Read Cell Values			
	11	10	00	01
11	0.00	18.39	0.03	4.01
10	0.07	0.00	62.22	1.84
00	0.00	0.06	0.00	13.39
01	0.00	0.00	0.00	0.00

(B)

Frame Length, N = 8192				
P/E Cycles	Lower Page		Upper Page	
	Mean	Variance	Mean	Variance
6000	14.85	29.64	7.18	10.23
8000	30.03	84.81	14.46	24.37
10000	52.61	216.95	26.06	51.30

Table I(A) shows the frequency of cell errors measured as a percentage of total number of cell errors observed across all P/E cycles. We observe that there is asymmetry in the number of $0 \rightarrow 1$ and $1 \rightarrow 0$ bit errors in the lower page and, in the upper page; the degree of asymmetry is much lower. The number of bit errors per frame parameter is the key factor in determining the ECC FER performance. From P/E cycling experiment data, we obtain the sample mean and variance statistics of the number of bit errors per frame in both lower and upper pages as shown in Table I(B). We observe that the variance in the number of bit errors per frame is much larger than the mean i.e., the experiment data is overdispersed with respect to a binomial distribution $B(n, p)$ typically used to model count data whose variance ≈ mean when p is small.

IV. CHANNEL MODELS FOR MLC FLASH MEMORIES

Based on the error characterization, we study a per-page BAC model and propose a per-page binary discrete channel model based on the beta-binomial probability distribution (2-BBM channel model) for MLC flash memories. Using empirical results and analysis, we show that the 2-BBM channel model is suitable for ECC FER performance estimation.

A. Definitions and Notation

Let K represent the total number of bit errors in a frame of length N bits. Let K_m be the total number of bit errors in a frame of N bits which consists of exactly m zeros. We denote the number of $0 \rightarrow 1$ and $1 \rightarrow 0$ bit errors in a frame with m zeros by $K_m^{(0)}$ and $K_m^{(1)}$ respectively. We have

$$K_m = K_m^{(0)} + K_m^{(1)} \tag{1}$$

where $K_m \in \{0,1,\ldots,N\}$, $K_m \in \{0,1,\ldots,m\}$ and $K_m \in \{0,1,\ldots,N-m\}$. K can also be represented as the sum of the total number of $0 \rightarrow 1$ and $1 \rightarrow 0$ errors as

$$K = K^{(0)} + K^{(1)} \tag{2}$$

$$\Pr\left(K^{(u)} = k\right) = \sum_{m=k}^{N} \frac{\binom{N}{l}}{2^N} \Pr(K_m^{(u)} = k) \tag{3}$$

where $u \in \{0,1\}$ and $l = m + (N - 2m)u$.

B. The 2-Binary Asymmetric Channel (2-BAC) Model

For the BAC model, $K_m^{(0)}$ and $K_m^{(1)}$ are distributed as per the binomial probability distribution and are independent i.e.,

$$K_m^{(0)} \sim B(m,p); \quad K_m^{(1)} \sim B(N-m,q); \quad K_m^{(0)} \perp K_m^{(1)}$$

where p and q denote the probabilities of $0 \rightarrow 1$ and $1 \rightarrow 0$ bit errors respectively and $B(m,p)$, $B(N-m,q)$ denote binomial probability distributions.

Proposition 1: The mean of K ($E[K]$) and the variance of K ($Var[K]$) for a BAC model are given by

$$E[K] = \frac{N}{2}(p+q) \tag{4}$$

$$Var[K] = \frac{N}{2}((p+q) - pq - \frac{1}{2}(p^2+q^2)) \tag{5}$$

The parameters of the BAC model p and q are estimated as the average $0 \rightarrow 1$ and $1 \rightarrow 0$ bit error rates obtained empirically. From proposition 1, $Var[K] < E[K]$ and for small values of p and q, $Var[K] \approx E[K]$. Thus the 2-BAC model of a MLC flash

memory does not fit the empirical observations in Table. I(B) and hence is unsuitable for accurate ECC FER performance estimation. However, note that the 2-BAC model does provide an accurate estimate of the average RBER which is given by $\frac{E[K]}{N}$. This shows that the ability to accurately estimate/predict the average RBER is not the sole criterion for a good MLC flash memory channel model.

C. The 2-Beta-Binomial (2-BBM) Channel Model

From the empirical error characterization results, we observed that the mean and the variance statistics of the number of bit errors per frame indicate overdispersion with respect to a binomial distribution. The beta-binomial probability distribution was first proposed in [3] as the probability distribution of counts resulting from a binomial distribution if the probability of success varies according to the beta distribution between sets of trials. Using empirical data, it was also shown in [3] that the beta-binomial probability distribution is suitable for modeling overdispersed count data. For the beta-binomial (BBM) channel model corresponding to the lower/upper page, we model the variables $K_m^{(0)}$ and $K_m^{(1)}$ as being distributed according to the beta-binomial distribution i.e.,

$$p \sim f(a,b); \quad K_m^{(0)} \mid p \sim B(m,p); \quad K_m^{(0)} \sim B_f(m,a,b)$$

$$q \sim f(c,d); \quad K_m^{(1)} \mid q \sim B(N-m,q); \quad K_m^{(1)} \sim B_f(N-m,c,d)$$

where $f(a,b)$, $B(m,p)$ and $Bf(m,a,b)$ denote the beta, binomial and the beta-binomial probability distributions respectively (similarly $f(c,d)$, $B(N-m,q)$ and $Bf(N-m,c,d)$). $X \mid Y$ denotes "X given Y". The parameters a,b,c,d of the BBM channel model are estimated from the sample moments of $K^{(0)}$ and $K^{(1)}$ using the method of moments [3]. From empirical data, we obtain the sample mean and second moment estimates of $K^{(0)}$ and $K^{(1)}$. Let μ_1, μ_2 represent the first and second moment estimates of $K^{(0)}$ (similarly μ_3, μ_4 for $K^{(1)}$). The parameter estimates are given by

$$\hat{a} = \frac{\mu_1^2(N+1) - 2\mu_1\mu_2}{N(\mu_2 - \mu_1) - \mu_1^2(N-1)}; \quad \hat{b} = \hat{a}\left(\frac{N}{2\mu_1} - 1\right) \tag{6}$$

$$\hat{c} = \frac{\mu_3^2(N+1) - 2\mu_3\mu_4}{N(\mu_4 - \mu_3) - \mu_3^2(N-1)}; \quad \hat{d} = \hat{c}\left(\frac{N}{2\mu_3} - 1\right) \tag{7}$$

D. ECC FER Performance Estimation Results

Fig. 1 shows the FER performance of a BCH code and a regular quasi-cyclic LDPC (QC-LDPC) code with $d_c = 64$ and $d_v = 4$ where d_c and d_v refer to the check

node and variable node degrees respectively in the parity check matrix. For both the ECCs considered, we observe that the 2-BAC model provides an optimistic estimate of the FER performance when compared to the empirically observed FER performance. This is mainly due to the inability of the 2-BAC model to capture the high variance in the number of bit errors per frame observed empirically. From these results it is clear that the proposed 2-BBM channel model is able to accurately describe the nature of the number of bit errors per frame in MLC flash memories and hence provides accurate estimates of the ECC FER performance.

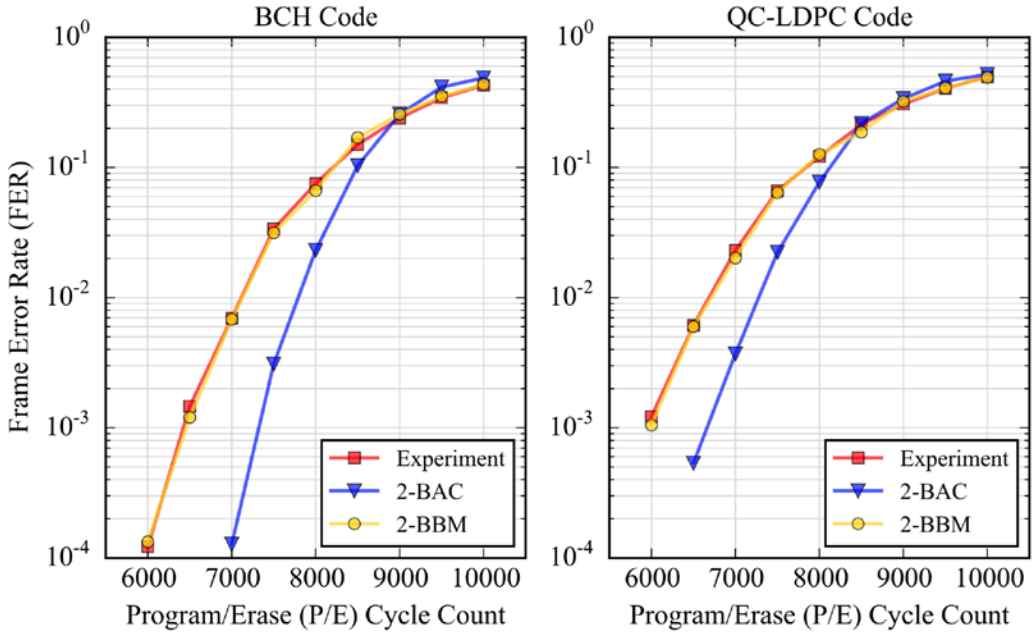

Figure 1: Comparison of FER performance of a (N = 8191, k = 7683, t = 39) BCH code and a (N = 8192, k = 7683) regular QC-LDPC code using empirical error data and error data from simulation using the 2-BAC and the 2-BBM channel models.

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