# MONTE CARLO CALCULATION OF MAGNETIC ANHYSTERESIS

H. N. BERTRAM
Ampex Corporation, Redwood City, California

**Résumé.** — Nous calculons la magnétisation anhystérétique d'un ensemble de particules alignées, de forme ellipsoïdale, constituées de domaines simples, identiques dans le cas de champs appliqués coïncidant avec la direction de l'alignement. Dans ce modèle nous déterminons la polarité finale de chaque moment, en avançant, pas à pas, au hasard à travers l'ensemble et en utilisant le simple critère qui fait appel seulement au champ continu et aux champs d'interaction. Les courbes relevées dépendent uniquement de l'ellipticité de la particule et de la structure géométrique de l'ensemble ; elles se comparent bien aux valeurs mesurées.

**Abstract.** — The anhysteretic magnetization of an assembly of aligned, ellipsoidal, identical single domain particles is calculated for the case of the applied fields coincident with the alignment direction. In this model, the final polarity of each moment is determined by proceeding randomly through the assembly and using a simple criterion which invokes only the dc and interaction fields. The resultant curves depend solely on particle ellipticity and assembly structural geometry and compare well with measured values.

Anhysteretic magnetization is achieved by ac demagnetization in the presence of a small dc field. When this process is applied to assemblies of single domain particles (SDP) it is closely related to ac bias tape recording. Any plausible theory of anhysteresis must include the interparticle dipolar interaction fields, otherwise the initial susceptibility is infinite [1]. It is argued here that a further essential factor in the proper characterization of the process for these systems is the spatially random order in which the final direction of each particle moment is determined. The calculation presented demonstrates how a simple model may be used to relate the primary structure factors of SDP assemblies (i. e., volume packing fraction, particle size and shape) to the anhysteretic magnetization.

Consider an assembly of identical, uniaxially anisotropic, prolate ellipsoidal, single domain particles situated in a specific geometric arrangement. All anisotropy axes are taken to be parallel while only applied fields and components of the interaction fields along the alignment direction are considered. In this calculation the statistical nature of the anhysteretic process is exploited in detail, since (for each value of dc field) the final polarity of every moment is determined. As the ac field is lowered, it is assumed that each moment ceases switching sequentially. The final polarity of each moment is determined by a simple criterion which involves only the dc and interaction fields acting on the particle. These fields offset the individual particle's loop with respect to the ac field ; the final polarity is related solely to the sign of the offset. The ac field (assuming an infinitesimally small decrement) does not affect the final polarities ; it serves only to provide the switching energy. In fact, the criterion hinges only on the existence of a hysteretic loop for each particle. Thus, to first order, the anhysteretic process is independent of assembly coercivity, individual moment switching mechanism and magnitude.

The assumption of a spatially random, sequential switching field order is fundamental to the anhysteretic process. Interactions can induce complex collective processes during moment reversal in SDP assemblies [2]. However, measured major loops and concomitant switching field distributions in magnetic tapes and powders never show extreme collective behavior. The remanence switching field distribution is always quite wide with a « half-height » width only slightly less than the remanence coercivity for a wide range of coercivities [3]. This width is most likely caused by distributions in intrinsic particle properties. Since SDP assemblies are formed from a random mixture of particles, the intrinsic, non-interacting, particle switching field magnitudes are distributed at random from particle to particle. It is logical, therefore, to assume that while the presence of interactions may alter the switching order, the moments, in any reversal process, continue to switch, in an approximately random sequence throughout the assembly.

The offset of an individual particle's loop is simply the ac field magnitude necessary to switch the particle in one direction minus the corresponding magnitude in the other. The dc field contributes twice its value to this shift. The interaction field contribution consists of two parts : the interaction fields from those « harder » particles in the sequence which have had their final polarization already determined and the fields from those « easier » moments which are still reversing. In this aligned model the « harder » particles are static and contribute (similar to the dc field) twice their interaction field to the loop offset. The « easier » particles, since they are still reversing with the ac field, do not contribute to this offset. Thus, the simple rule applied to each moment is

$$H_{dc} + \sum H_i \gtrless 0 \Rightarrow \text{final polarity} \pm$$

where the interaction term ($H_i$) is summed over those particles no longer reversing ; +, − correspond to polarities along or opposite to the dc field respectively. In reality SDP tape assemblies are not perfectly aligned ; however, to first order, the interaction contribution is unchanged. This model is to be contrasted with a previous theory in which only the interaction field due to reversible rotations was considered [4].

The calculation is performed conveniently by a large memory computer. For each chosen value of the dc field the computer goes first to the particle labeled

No. 1 ; since there are no static interaction fields yet, it sets the polarity of moment No. 1 along the dc field. Next, it moves to particle No. 2, situated randomly elsewhere in the assembly. The dc field and interaction field from moment No. 1 are added and the rule applied to yield the final polarity of moment No. 2. Moving to particle No. 3, the interaction field from No. 1 and No. 2 are added to the dc field and again the rule is applied to yield the polarity of moment No. 3. The process is continued through the assembly until a net magnetization is completely acquired.

A SDP assembly was simulated by placing 1 000 particles of ellipticity $\varepsilon$ on a $10 \times 10 \times 10$ lattice such that the particles were in contact in the field direction and were separated by equal amounts ($\varepsilon/\alpha$) in the two transverse directions (Fig. 1). Each particle

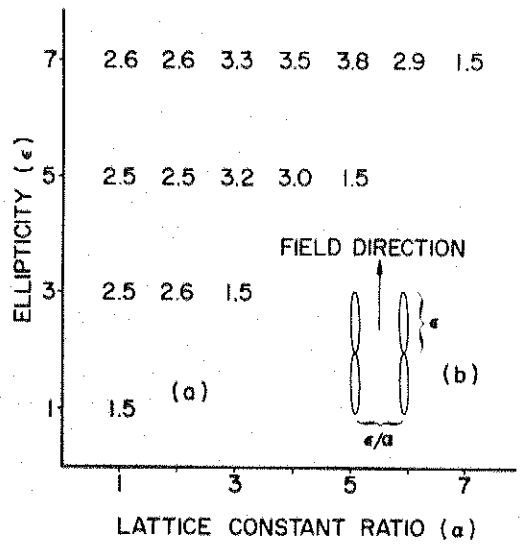


FIG. 1. — a) Initial anhysteretic susceptibility for various particle ellipticities and transverse particle spacings. b) Illustration of particle configuration used in the calculation.

was assigned a number at random (from 1 to 1 000) in order to allocate the switching sequence. The interaction field between any two moments was taken as the field at the center of one moment due to the fully magnetized ellipsoidal body of the other. For ease of computation, the magnetic field from an ellipsoid of revolution of ellipticity $\varepsilon$ was approximated by appropriately stacking ($\varepsilon$) dipoles.

The calculations yielded the entire magnetization curve. The curves were found to be independent of the particular sequence as long as it was random. Suitable approximation techniques coupled with periodic boundary conditions were used to « extend » the $10 \times 10 \times 10$ assembly to any desired external sample shape and ensure that the process was not dominated by the moments on the surface. It is illustrative to present initial susceptibilities for various assemblies with no external shape demagnetization. In figure 1, these susceptibilities are given for $\varepsilon = 1, 3, 5, 7$ and $\alpha = 1, 2, ..., \varepsilon$. Commercial $\gamma Fe_2O_3$ tapes with 5 : 1 ellipsoidal particles and volume packing fractions of $\rho \cong .35$ have initial susceptibilities along the alignment direction of $\chi \cong 2.0$. This corresponds to $\varepsilon = 5$, $\alpha = 4$, since $\rho \cong 1/2\ (\alpha/\varepsilon)^2$. The comparison is quite good if one (reasonably) scales the aligned susceptibility calculated here ($\chi \cong 3.0$) down by the tape squareness ratio (= .75).

There are several important generalizations about the anhysteretic process which follow from the model presented here.

1) The model emphasizes the playoff of applied field versus the interaction fields so that, to first order, the anhysteretic magnetization is independent of assembly coercivity.

2) Since the criterion which determines the final orientation of each moment is a linear sum of applied and interaction fields and since both interaction fields and magnetizations scale with particle moment, the initial susceptibility is independent of moment.

3) Since only the physical size, shape, and packing of the particles enter the calculation, only those changes in structural geometry affect the curves. However, geometric scaling of an assembly does not constitute a significant change ; for example, spherical particles on a cubic array (akin to a random assembly) have identical initial anhysteretic susceptibilities as the volume packing fraction is increased to the point of interparticle contact.

The calculation which has been presented here is attractive primarily because it exposes the simplicity of the anhysteretic process. The theory is also quite general since various collective effects may be easily included. For example, the word « particle » used here can be taken to represent as well a tightly bound bundle of particles in order to simulate clumping. In fact, due to this equivalence, particle clumping should not seriously alter the anhysteretic process. Finally, the model may be used to calculate the anhysteretic magnetization for a wide variety of particle configurations.

## References

[1] WOHLFARTH (E.), *J. Appl. Phys.*, 1964, **35**, 783.
[2] BERTRAM (H.) and MALLINSON (J.), *J. Appl. Phys.*, 1969, **40**, 1301.
[3] GUSTARD (B.), (Private Communication).
[4] KNELLER (E.), *J. Appl. Phys.*, 1968, **39**, 945.