# NOTES ON THE PROBLEM OF MAGNETIZATION REVERSAL

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## 1. Introduction

A perennial non-linear problem in magnetic storage technology is the problem of magnetization reversal. Its complete analytic solution has never been accomplished, and heavy reliance has had to be placed on computer simulations. Although these really are a type of experiment, they furnish some insight into the switching process, simply because they allow one to look at the motion of computer 'spins' as a function of time and of position, whereas the motion of the actual physical spin system can at best be studied by inference from data of limited resolution. Nevertheless, the computer simulations do not tell us why the spins move the way they do. They only help us to dismiss model theories that do not agree with them. The need for a good theory is becoming more and more apparent as magnetic recording pushes towards higher and higher data rates (a nanosecond time scale is the immediate goal), and ever higher recording densities.

Since an all-encompassing, 'heroic' solution of this problem is still out of the question, we must content ourselves with attacking the problem piece by piece. For example, before attacking the dynamics, we may try to determine the static energy landscape of the spin system. For general values of the switching field below a critical value, reversal is presumably initiated by thermally activated escape over one or more energy barriers.

## 2. The Energy Landscape

We therefore begin by determining these barriers for a particularly simple case: a linear chain of spins, interacting via dipole forces only (exchange could be added without any significant modification of the results). To simplify the problem further, we restrict the interaction to nearest neighbors; however, we retain the full angular symmetry of the dipolar coupling. For simplicity, we restrict rotation of the spins to a plane. (This corresponds to thin films, or to very flat coplanar particles). The energy of such a chain is evidently



*R. Marcelli and S.A. Nikitov (eds.), Nonlinear Microwave Signal Processing: Towards a New Range of Devices*, 3–11.

$$E = \sum_i \sum_\delta [\cos(\theta_i - \theta_{i+\delta}) - 3\cos\theta_i \cos\theta_{i+\delta}] - H\cos\theta_i - ½K\cos^2\theta_i \quad (1)$$

where the δ-summation is over the two nearest neighbors of the $i^{th}$ spin. Also, *H, K* are respectively the applied field and the anisotropy field (shape or crystalline). These fields are measured in units of the dipole field, and distances are measured in units of spacing between the spins. For positive *K*, the easy axis is along the chain. The stationary states are obtained by equating all derivatives of *E* with respect to each $\theta_i$ to zero. This gives a set of nonlinear recursion relations:

$$2\sin\theta_i(\cos\theta_{i+1} + \cos\theta_{i-1}) + \cos\theta_i(\sin\theta_{i+1} + \sin\theta_{i-1}) + (H + K\cos\theta_i)\sin\theta_i = 0$$

Their solution requires the use of the computer, but only in a trivial way. Assuming the chain to be semi-infinite, the results are as follows:

## 2.1 SOLUTIONS THAT PERVADE THE ENTIRE CHAIN

All these are periodic (we have not found any chaotic solutions). There are l-cycle, 2-cycle, 3-cycle etc. solutions. By l-cycle is meant the solution with all cosθ equal (other than the solution θ=0, which is, of course, the ground state). There are two such solutions; one with all angles equal and in the same quadrant, the other with successive angles still having the same cosine, but with sinθ alternating in sign, so that successive angles are reflections of each other in the x-axis. For the first of these, we have is $\theta=\theta_a$ ,where

$$\cos\theta_a = -\frac{H}{6+K} \quad (3)$$

For the second one we have $\theta=\theta_b$, where:

$$\cos\theta_b = -\frac{H}{2+K} \quad (4)$$

For an isolated spin, these cosine would equal $-H / K$, and would give the angular position of the barrier height due to the anisotropy. Evidently, according to formula 3, the dipole interaction has increased the anisotropy from *K* to *K* + 6 and to *K* + 2 respectively. Substituting the result 3 in the formula for the energy gives the energy as a function of the fields (fig. 1 for formula 3, and fig.2 for formula 4). Evidently, the second case has lower energy.

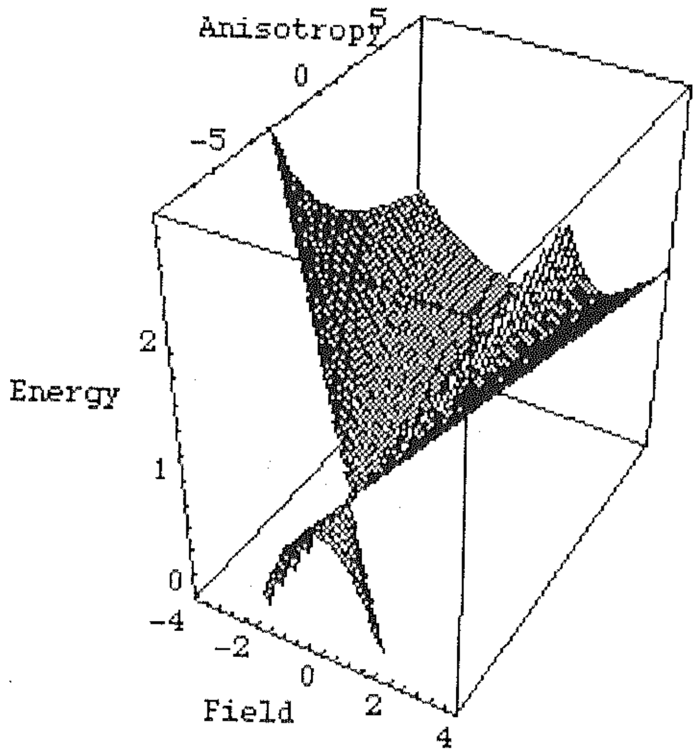


Figure 1

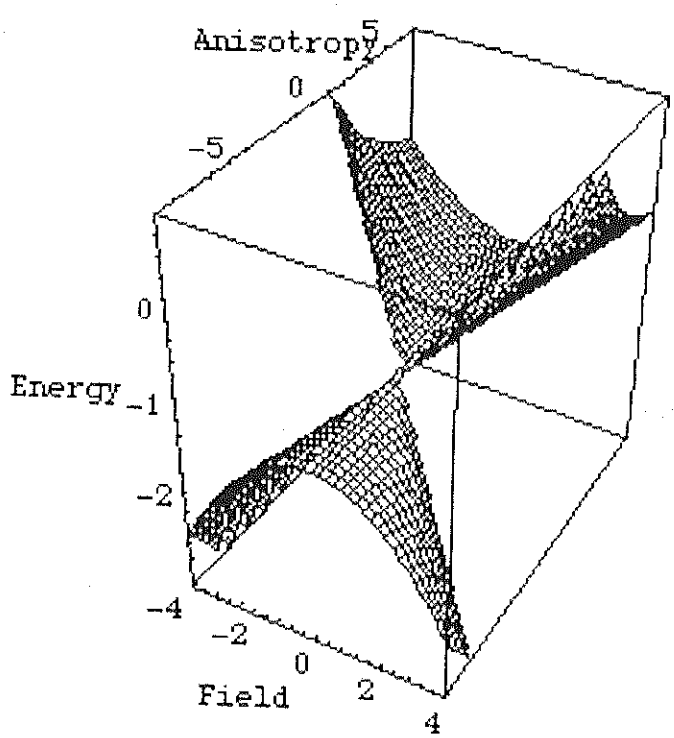


Figure2

For 2-cycle solutions, the value of the angle, as well as its cosine, alternates between twodifferent values $\theta_1$, $\theta_2$. Writing $\cos\theta_1=x$, $\cos\theta_2=y$, we can represent the fields parametrically as:

$$H = -4(x+y) + \frac{2xy}{y-x}\sigma_1\sigma_2\left(\sqrt{\frac{1-x^2}{1-y^2}} - \sqrt{\frac{1-y^2}{1-x^2}}\right) = (x+y)\left(-4 + \frac{2xy\sigma_1\sigma_2}{\sqrt{(1-x^2)(1-y^2}}\right)$$

$$K = 4 + \frac{2}{y-x}\sigma_1\sigma_2\left(x\sqrt{\frac{1-y^2}{1-x^2}} - y\sqrt{\frac{1-x^2}{1-y^2}}\right) = 4 - 2\frac{(1+xy)\sigma_1\sigma_2}{\sqrt{(1-x^2)(1-y^2}} \tag{5}$$

Here σ's is either +1 or -1 according to whether the sign of sinθ is the same or opposite to that of the cosine. Substitution in expression 1 then gives the energy surface in parametric form, shown for the case $\sigma_1\sigma_2=1$ in figure 3.

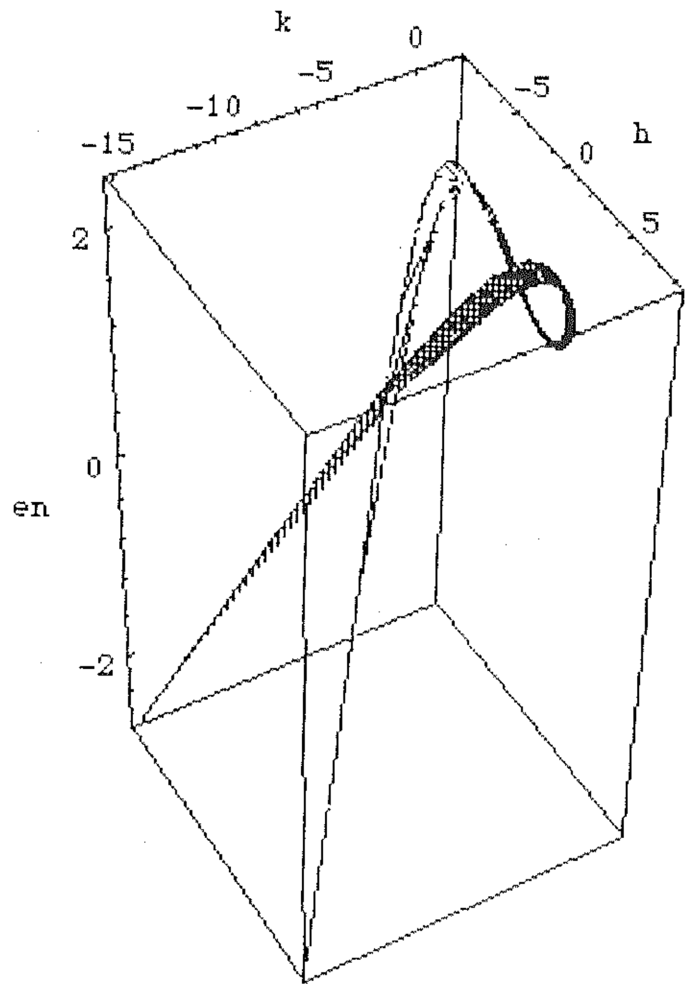


Figure 3

The case $\sigma_1\sigma_2$=-1 is shown in figure 4.

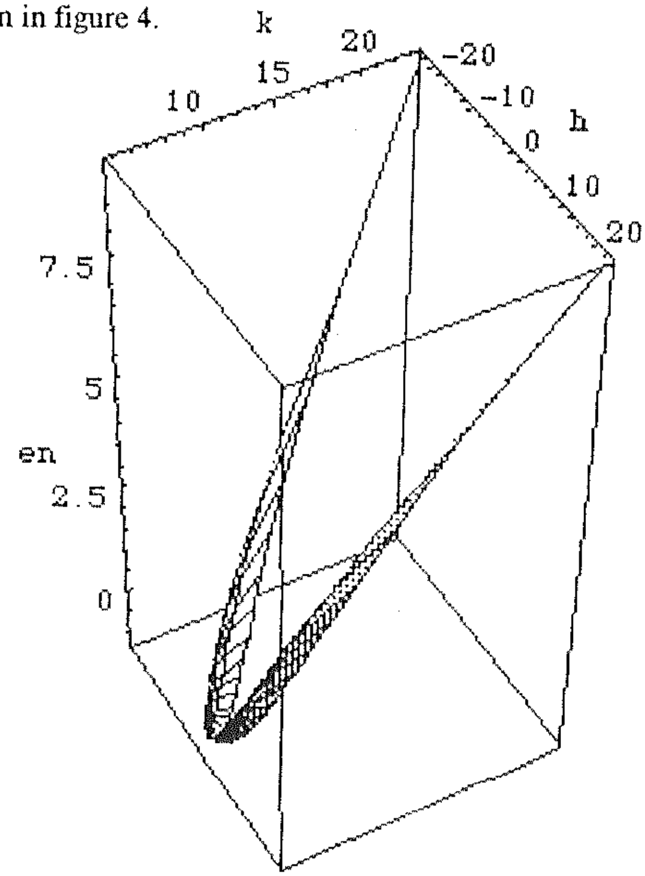


Figure4

3-cycles and higher cycles are much harder to evaluate. However, they all have more or less tha same ribbon like appearence in *E, H, K* space.
A calculation of the second derivatives of the energy, and solution of the secular equation shows that all the stationary states discussed here are saddle points of various orders and are therefore dynamically unstable. We have not been able to Prove this in the general case of the n-cycle, but feel that it is in fact true. This is important; it enables one to dismiss in this particular case the popular notion that a complex system has a very large number of metastable states, in which the system can get trapped and cannot reach the ground state within a reasonable time. However, if imperfections are admitted into the chain, metastable states might arise.

## 2.2 SOLUTIONS CONFINED TO THE BEGINNING OF THE CHAIN

In the infinite, uniform chain, only the stationary states enumerated above exist .
For a semi-infinite chain, however, there are also some edge-states at the beginning of the chain. These extend over only a few spins (typically two to ten for the cases we have studied numerically. They also differ from the extended states in that the angular

excursions are generally smaller. For modest deviations of two initial θ's from zero (two values are needed, since the recursion relation 2 is second order), the solutions for $\theta_i$ become complex for quite small values of the subscript $i$. Naturally the energy of these states, while still greater than the ground state energy, is smaller by a factor (length of chain)$^{-1}$ than the energy of the extended states. It is not easy to map out the basin of attraction within which the initial values $\theta_1$, $\theta_2$ must lie if these local states, rather than the extended states, are to be found. A crude idea of that basin can be formed by considering just the first non-trivial recursion relation and to determine in what region of the $\theta_1$, $\theta_2$ plane, $\theta_3$ becomes complex ( see fig 5). (The actual basin, of course, will be a bit bigger; in particular it will engulf the origin completely). The contours in figure 5 are labeled with the value of $\cos\theta_3$, the axes are $\theta_1$, and $\theta_2$,. Values of these variables within the black area belong only to localized states.
These results have a counterpart in the linearized approximation to equation 2. That linearized version is

$$\delta\theta_{n+2} + h\delta\theta_{n+1} + \delta\theta_n = 0$$

where

$$h = H + K + 4$$

The solution has the form

$$\delta\theta_n = A\gamma_+^n + B\gamma_-^n$$

where

$$\gamma_\pm = -\tfrac{1}{2}h \pm \sqrt{\tfrac{1}{4}h^2 - 1}$$

If $|h| < 2$, the γ's are unimodular complex numbers, and the stationary state oscillates and is extended. For $|h| > 2$, that is for $H > -K - 2$, or else for $H < -K - 6$, the disturbance decays along the chain (the growing root must be rejected for the usual reasons). In the full nonlinear theory this corresponds to the state localized at the beginning of the chain.

## 3. Relation to More Realistic Structures

One may hope that these conclusions have some limited relevance to switching of an actual magnetic sample. Thus, the analog of switching via the 1-cycle barrier state would be Stoner-Wohlfarth uniform rotation of the entire magnetization. The higher cycle states would correspond to switching via various buckling modes. In the present model, there is no analog of the curling modes, because spin rotation has been confined to a plane.

The analog of the localized states would be surface states in an actual sample. There, too, these states would present a much lower barrier to switching than the extended states, the 'rest' of the magnetization will then switch by front propagation (i.e. domain wall motion). These matters will be the subject of a forthcoming publicationl. Note that imperfections, just like the surface, can result in localized stationary states, so that a multi-barrier treatment is still indicated for samples with ample defects.

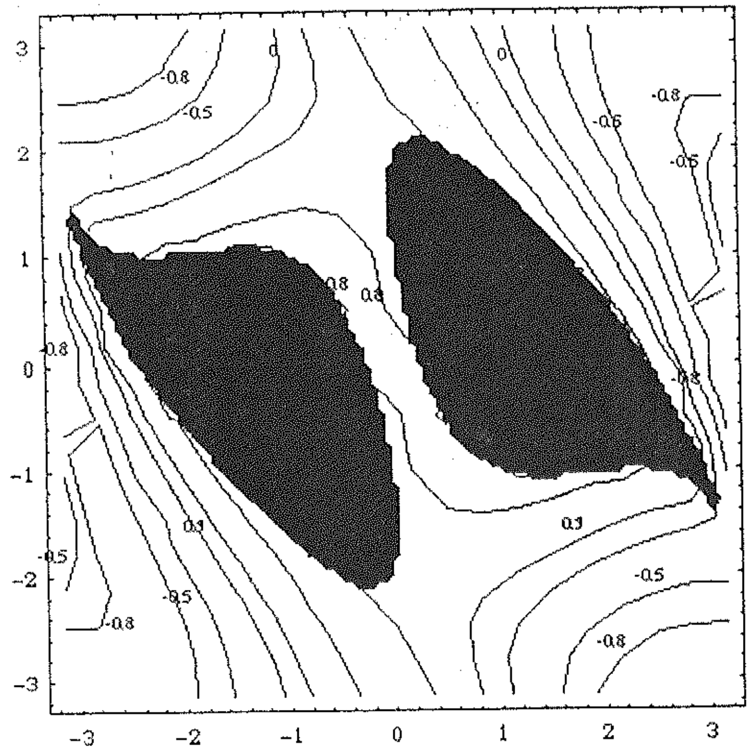


Figure 5

### 4. Switching in the Presence of Many Barriers.

Front propagation is probably not the rate-limiting step in the magnetization reversal of a realistic imperfect sample. Therefore it is reasonable to separately examine the question of switching impeded by many barriers.
For a single barrier of height $E$, the escape rate $v$ should obey an Arrhenius formula:

$$v = \omega\, e^{-E_B/kT} \tag{6}$$

where ω is some attempt frequency into whose nature we do not inquire at this point.
If $n_0, n_1$ are the probabilities of finding the system in a metastable state (O) to the ground state or another, lower, metastable state (l), the two master equations governing these two probabilities give for $n_0$, i.e. for the probability of not having switched after time $t$

$$n_0(t) = A\{e^{-(E_0-E_1)/kT} + \exp-(\nu t / A)\}, \quad A = (1+e^{-(E_0-E_1)/kT})^{-1} \tag{7}$$

This is also essentially the magnetization remaining in the initial direction after time $t$.
If $E_1 << E_0$, so that the reverse barrier is much higher than the forward barrier, this reduces to $e^{-\nu t}$. The observations seem to give a linear decay of the curve of $n_0$, versus log$t$ over several decades. Clearly a single exponential cannot deliver this result. To approximately fit the data, it has been suggested that several decay processes must be going on in parallel, so that one should write (in the case of high reverse barriers)

$$n_0 = \sum_k W_k e^{-\nu_k t} \tag{8}$$

all the weights $W_k$, being positive, and $\Sigma_k W_k = 1$. Accordingly,

$$\frac{dn_0}{d\log t} = -\ln(10)\sum_k W_k \nu_k t\, e^{-\nu_k t} \tag{9}$$

Unfortunately the absolute value of the right hand side has a maximum, which cannot exceed $e^{-1}\log(10) \cong 0.85$, because $xe^{-x}$ is maximal at $x = 1$, where its value is $e^{-1}$.
By contrast, the measurements reported in literature give a slope of about 1.4 near the coercive field, i. e. almost twice as high as the simple formula 8.
Two of the authors (R.A. and H.N.B.) have proposed that this difficulty may be overcome by a more detailed solution of the master equation that allows for the Dossibilitv that some of the barriers may have to be surmounted in series rather than in parallel. With barriers in series, the probability of not switching obviously becomes enhanced . The enhancement becomes relatively easy to calculate when the barriers are all equal, but are much higher for reverse than for forward motion (respectively away from, and towards, the global minimum). Further, detailed analysis shows that not only the probability of not switching, but also its maximum time rate of change is enhanced. This appears in the experiments as a steepening of the $dM/d\log t$ plot.

## 5. Relation of High Data Rate Recording to Front Propagation and Soliton-like Motion

Whereas in ordinary materials, front propagation may not be the rate limiting step, it is most certainly an upper limit on the rate for perfect magnetic systems. In view of current pressures to maximize recording rates it is therefore worth while to extend the above analysis. Extending it to higher dimensional arrays will involve greater computational effort, but will probably not produce major qualitative changes. Conceivably, some surprises will result from inclusion of precessional effects in the analysis. Perhaps the most important problem to be addressed will be the interaction of fronts under actual recording conditions, such as during inscription of di-bits, etc. Hopefully we will be able to profit from current knowledge of such nonlinear interactions in other areas of communications technology.