Transient Thermomechanical Contact of an Impacting Sphere on a Moving Flat

Contact between a slider and a magnetic recording disk is modeled as transient contact of a sphere on a moving flat. The sphere is assumed to be rigid, and the flat is treated as an elastic-plastic body with isotropic hardening. Heat generation is related to friction at the contact interface. Dimensionless solutions are obtained for maximum temperature rise, maximum contact force, maximum contact area, and maximum penetration as a function of dimensionless vertical initial velocity of the sphere. It is observed that transient thermomechanical contact with elastic-plastic deformation deviates from “classical theories” for dynamic elastic and quasi-static elastic-plastic contacts as the dimensionless vertical initial velocity of the sphere increases. The results are applied to optimize the slider-disk interface in a hard disk drive with respect to slider-disk contacts. [DOI: 10.1115/1.4003996]

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1 Introduction

The understanding of deformation and temperature rise in sliding contacts is important for the design of mechanical components. The transient contact between a slider and a rotating disk in a hard disk drive is a typical example of a sliding contact. Undesirable erasure of information can occur during slider-disk contacts [1]. The loss of data is strongly dependent on the stress and the temperature reached inside the magnetic recording film on the top surface of the disk [2].

A number of investigations have been published in the last few years that analyze the effect of various design parameters on the mechanical and thermal response of contacting bodies [3–27].

Kral and Komvopoulos [3] used a three-dimensional quasi-static finite element model to investigate strain and stress fields of a layered flat in contact with a rigid sphere under normal and tangential loading. Tao et al. [4] studied the transient three-dimensional contact between a two-rail slider and a rotating disk in a hard disk drive. In Ref. [5], a plane strain finite element model of a rod sliding on a layered flat was analyzed. The coefficient of restitution for an oblique sphere-flat contact at high impact velocities was studied in Ref. [6] using three-dimensional finite element analysis. Katta et al. [7] investigated the effect of sphere radius and normal impact velocity on maximum penetration, mean contact pressure, and contact force using Hertz’s classical theory combined with a quasi-static elastic-plastic contact model.

Tian and Kennedy [8] developed analytical and approximate solutions for the maximum and average surface temperature rise resulting from a square, circular, and parabolic heat source. A numerical algorithm for steady-state heat partitioning and the associated flash temperatures was presented in Ref. [9] for arbitrarily shaped contacts. A three-dimensional numerical model was developed in Ref. [10] to study the effect of roughness and sliding speed on the transient temperature rise. Laraqi [11] developed an analytical solution for the steady-state surface temperature of a circular heat source sliding over a half-space showing good agreement with Ref. [8]. Wen and Khonsari [12] included convective cooling of the half-space surface subjected to frictional heating. Another algorithm for predicting the steady-state temperature rise resulting from frictional heating within an interface of two sliding bodies was described in Ref. [13].

Kulkarni et al. [14] used the finite element method to study a two-dimensional thermomechanical model of a rolling-sliding contact in the elastic regime of deformation. The finite element model described in Ref. [14] was extended to a kinematically hardening material with temperature dependent yield strength [15]. Wang and Liu studied a two-dimensional thermomechanical asperity contact model with steady-state heat transfer [16]. Three-dimensional thermomechanical models were developed for the contact of nominally flat [17] and rough surfaces [18] assuming steady-state heat transfer. Studies were also performed to include frictional heating between rough contacting surfaces [19] and temperature dependent yield strength material [20]. Thermo-elastic stress fields were obtained in a half-space subject to frictional heating [21]. Ye and Komvopoulos [22] performed a three-dimensional finite element analysis of an elastic-plastic layered and patterned [23] media subjected to frictional heating due to a sliding sphere under constant normal load and tangential velocity. The effect of material properties and layer thickness on temperature and stress and strain evolution was investigated. A three-dimensional thermomechanical model applicable for rolling and sliding contacts was developed in Ref. [24] assuming small deformations and steady-state heat transfer. An elastic contact mechanics analysis was developed in Ref. [25] between an impacting sphere and a flat. A flash temperature analysis was conducted based on the solution of Tian and Kennedy [8]. The model in Ref. [24] was improved in Ref. [26] by taking into account temperature dependent strain hardening and heat flux partitioning at the sliding interface. A two-dimensional thermomechanical finite element analysis of a rigid fractal surface sliding over an elastic or elastic-plastic medium was conducted in Ref. [27].

From the above literature survey it can be seen that most investigations were concerned with pure mechanical [3–7] or thermal [8–13] analysis. In addition, thermomechanical studies [14–27] using finite element and semi-analytical methods were performed. The latter studies were limited to either quasi-static (no inertia effects) or steady-state thermomechanical solutions between contacting bodies. No investigation is available in the open literature on the thermomechanical analysis of a transient contact between a sphere and a flat.

The goal of this paper is to study the thermo-elastic-plastic transient contact between a rigid impacting sphere and a deformable moving flat. The effect of impact conditions, material properties,
2 Modeling

2.1 Theoretical Model. Figure 1 presents a model of a rigid sphere impacting a moving flat. The velocity of the sphere is denoted by \( V_s \) and the velocity of the disk is \( V_f \) [2]. Additional details of the theoretical background can be found in Appendix A.1.

2.2 Material Properties and Constitutive Models. The material of the sphere and the flat is assumed to be homogeneous with material properties independent of temperature. The sphere is assumed to be rigid. The flat is deformable and can undergo elastic-plastic deformation.

A thermo-elastic-plastic material model was used as described by Hallquist [28]. This model allows the use of temperature dependent material properties. Stress is calculated based on elastic and thermal strain. When treating plasticity, the stress is updated elastically and checked whether it exceeds the isotropic yield function

\[
\phi = \frac{1}{2} S_{ij} S_{ij} - \frac{\sigma_y(T)}{3}
\]

where \( S_{ij} \) is the deviatoric stress tensor and \( \sigma_y \) is the yield strength in uniaxial tension. If the stress exceeds the elastic limit, the stress deviators are scaled back by a factor \( f_s \), i.e., \( S_{ij}^{\text{eff}} = f_s S_{ij} \) where the factor \( f_s \) is defined as

\[
f_s = \sqrt{\frac{2 G f_r}{S_{ij} S_{ij}}} \quad (2)
\]

The plastic strain is updated by the increment

\[
\Delta e_{\text{pl}}^{\text{eff}} = \frac{(1 - f_s) \sqrt{2 G f_r}}{G + 3 E_p} \quad (3)
\]

where \( G \) is the shear modulus and \( E_p \) is the plastic hardening modulus, assumed to be equal to 2% of the Young’s modulus.

Strain rate effects were not considered in the present paper. However, our model can easily be extended to take into account strain rate effects in further investigations.

2.3 Finite Element Model. Figure 2 presents the three-dimensional finite element model of a rigid sphere with radius \( R \) impacting on a moving flat. The finite element model is similar to the model developed in Ref. [2]. The summary of the main features of the finite element model is provided below.

The deformable flat is assumed to be a rectangular parallelepiped with dimensions from \( 7R \) to \( 20R \) in the \( x \) direction and \( 1.5R \times 1.5R \) in the \( y \) and \( z \) directions, respectively. The size of the flat was chosen to allow inclusion of stress and temperature fields inside the contacting bodies without causing edge effects. The flat moves with a constant tangential velocity \( V_f \). The sphere has a vertical initial velocity \( V_s \), which changes during the transient contact due to elastic-plastic deformation.

The \( xy \) plane is the plane of symmetry, i.e., we consider only one half of the hemisphere and the flat as shown in Fig. 2. The boundary conditions consist of displacement constrains in the \( y \) and \( z \) directions at the bottom of the flat (\( xz \) plane). Displacements are restricted in the \( z \) direction in the \( xy \) plane of symmetry. The top surface of the flat is free of traction except for the traction imposed by the impacting sphere. The top surface of the hemispher (\( xz \) plane) can move only in the \( y \) direction. To maintain pure vertical motion of the top surface of the sphere, the nodes at the top surface are coupled, i.e., the \( y \) displacements are forced to be equal. A uniform initial temperature is applied to both the sphere and the flat. Convection and radiation are neglected in this study. To fulfill the Block postulate [29] and provide an appropriate heat partitioning factor at the contact interface, the temperature of the sphere and the flat is assumed to be the same at opposing contacting nodes. This assumption was accomplished by allowing perfect thermal conductivity at the contact interface.

Additional details of the finite element model along with its validation can be found in Appendices A.2 and A.3, respectively.

3 Background for Analytical Solutions

3.1 Background of Dynamic Hertz Solution and Critical Values. The total contact time is given by [30] as

\[
t_{\text{tot}} = 2.87 \left( \frac{m^2}{RE^2 V_s} \right)^{1/5} \quad (4)
\]

where \( m \) is the mass of the sphere and \( E \) is the equivalent modulus of elasticity given by

\[
\frac{1}{E} = \frac{1 - v^2}{E_s} + \frac{1 - v^2}{E_f} \quad (4a)
\]

In Eq. (4a) subscripts \( s \) and \( f \) denote the sphere and the flat, respectively. The maximum penetration of the sphere into the flat is given by [30]

\[
\omega_{\text{max}} = \left( \frac{15 m v^2}{16 \pi^2} \right)^{2/5} \quad (5)
\]
Brizmer et al. [31] developed expressions for the critical values marking the first occurrence of plastic yielding at the transition from the elastic to the elastic-plastic regime of deformation in quasi-static sphere-flat normal contact. The critical interference (penetration) is given by

\[ \omega_c = \left[ C_v \frac{\pi}{2} \frac{Y}{E} \right]^2 R \]  

where \( \nu \) and \( Y \) are the Poisson’s ratio and yield strength of the softer material (flat in this study), respectively, while

\[ C_v = 1.234 + 1.256 \nu \]  

The critical contact force is defined by

\[ P_c = \frac{\pi^3}{6} c^3 Y \left( \frac{R}{E} \right)^2 \]  

and the critical contact area is given by

\[ A_c = \pi \left[ C_v \frac{\pi}{2} \frac{Y}{E} \right] R \]  

Combining Eq. (5) with Eq. (6), we can derive the critical vertical initial velocity

\[ V_{y, \omega} = \sqrt{\frac{16}{15m} \left( C_v \frac{\pi}{2} \frac{Y}{E} \right)^5 E R^3} \]  

Based on Eqs. (4) and (9) the critical contact time can be calculated as

\[ t_c = 2.87 \left( \frac{m^2}{RE^2 V_{y, \omega}} \right)^{1/5} \]  

### 3.2 Background of Tian and Kennedy Thermal Solution

Tian and Kennedy [8] developed an approximate analytical solution for the steady-state maximum temperature rise of a sphere sliding on a flat in the elastic regime of deformation. Normal load and sliding velocity are assumed to be constant during the contact between the sphere and the flat. The steady-state maximum temperature rise at the contact interface is given by Tian and Kennedy [8] as

\[ \Delta T_{\text{max, jK}} = \frac{1.31 \mu p V_s}{k_f \sqrt{1.2344 + P e_f + k_s \sqrt{1.2344} + P e_s}} \]  

where \( a \) is the contact radius, \( \mu \) is the coefficient of friction, \( p \) is the mean contact pressure, and \( k_f \) and \( k_s \) are the thermal conductivities of the flat and the sphere, respectively. In addition, \( P e \) is the Péclet number defined as

\[ P e = \frac{V_s a}{2K} \]  

where \( K \) is the thermal diffusivity

\[ K = \frac{k}{\rho c} \]  

and \( \rho \) and \( c \) are the material density and the specific heat, respectively. We can define the critical maximum temperature rise based on Eq. (11) and using Eqs. (7) and (8) we can calculate to calculate the critical contact radius \( a_c = (A_c/\pi)^{0.5} \) and the critical mean contact pressure \( p_c = P_c/A_c \). Since \( P e_s = 0 \), the critical maximum temperature rise is given by

\[ \Delta T_{\text{max, s}} = \frac{1.31 \mu p V_s}{k_f \sqrt{1.2344 + P e_f + k_s \sqrt{1.2344}}} \]  

### 4 Numerical Results and Discussion

The parameters of main interest resulting from sphere-flat contacts are temperature rise, contact force, and area, as well as penetration. In this paper, we intend to obtain dimensionless solutions for the maximum values of these parameters. Dimensionless contact parameters can be used for the design and analysis of sphere-flat contacts. In particular, they can be applied for the optimum design of the slider-disk interface to reduce thermal erosion and physical damage to the magnetic medium [2].

For the results shown in Figs. 3–9, we have used the following values representing a typical case of slider to disc contact [2]: \( P e_{\text{max}} = 0.32, V_s/V_{y, s} = 157.3, E_s/Y_s = 106, \nu = 0.33, \) and \( \mu = 0.3 \). The maximum Péclet number is defined as

\[ P e_{\text{max}} = \frac{V_s \sqrt{A_{\text{max}}/\pi}}{2K} \]  

where \( A_{\text{max}} \) is the maximum contact area during transient contact.

Figure 3 shows the evolution of the dimensionless vertical velocity and the dimensionless contact area as a function of the dimensionless time. We observe that the dimensionless vertical initial velocity decreases during the contact from an initially negative value of \(-157\), reaches zero, and rebounds to a value of 185. The rebounding of the sphere is due to the elastic energy stored in the flat and the component of the friction force acting in the vertical direction at the leading edge of the contact interface. The dimensionless contact area shown in Fig. 3 between the sphere and the flat increases, reaches a maximum, and decreases thereafter. The maximum of the contact area occurs at the maximum penetration of the sphere into the flat.

Figure 4 shows the dimensionless vertical displacements as a function of the dimensionless time for a point fixed at the bottom of the sphere and a point fixed on the surface of the flat. The location of the point on the flat surface corresponds to the maximum penetration of the sphere as shown in Fig. 5. The dimensionless vertical displacement of the bottom of the sphere, marked by the dashed curve in Fig. 4, follows the change in the dimensionless vertical velocity shown in Fig. 3, i.e., the sphere penetrates up to the maximum depth of \( y/\omega = 65 \) around \( t/t_c = 0.22 \) and then returns back to zero after complete separation at \( t/t_c = 0.42 \). When the fixed point on the moving flat is far from the impacting sphere, its displacement is zero up to a dimensionless time of approximately 0.1.
As the fixed point approaches the impacting sphere, it deflects below the original surface profile \( y/c = 0 \) and then rapidly reaches its maximum value as a result of pile-up at the leading edge of the contact zone (see Fig. 5). Later on, when the bottom of the sphere passes the fixed point on the moving flat, it deflects below the original flat surface profile and the dimensionless vertical displacement value coincides with the sphere maximum penetration.

After the fixed point on the moving flat has passed the sphere completely, its vertical displacement recovers due to elastic recovery of the flat. It can be seen from Fig. 4 that the final dimensionless deflection of the fixed point on the flat is \( y/c = 0.54 \). This value corresponds to the residual maximum penetration depth on the flat surface \( y_{res\_max} \).

Figure 5 shows the plastic strain in the contact zone at the dimensionless time \( t/t_c = 0.22 \) corresponding to the maximum penetration. We observe that pile-up occurs at the leading edge of the contact due to plastic deformation of the contact zone.

Figure 6 shows the evolution of the dimensionless contact force and the dimensionless friction force during a contact. The friction force \( Q \) is composed of two terms. The first term corresponds to pure sliding friction at the contact interface. The second term is plowing friction due to the penetration of the sphere into the flat.

We observe from Fig. 6 that the profiles of the dimensionless contact and friction forces are asymmetric with respect to their maximum values.

In Fig. 7 the total friction coefficient \( \mu_{tot} = Q/P \) is plotted as a function of the dimensionless time. At the very beginning of the contact, the friction coefficient equals 0.3. This value of \( \mu_{tot} \) represents the sliding friction coefficient, which is assumed to be constant during the numerical simulation. Due to the penetration of the sphere into the flat and the resulting plastic deformation, the total friction coefficient during the impact deviates from the original value of 0.3 and rises to a maximum value of 0.46 at \( t/t_c = 0.24 \). During the rebounding of the sphere, the penetration depth and the total friction coefficient decrease. The final value of the friction coefficient equals 0.3 corresponding to pure sliding friction. The mean contact pressure \( p \), defined as the ratio of the contact force \( P \) and the contact area \( A \) normalized by the yield strength \( Y \) of the soft flat, is shown by the solid curve in Fig. 7. If the normalized mean contact pressure exceeds 1.07, plastic deformation occurs in the contact zone [30] as can also be seen in Fig. 5.

Figure 8 shows the evolution of the dimensionless temperature rise for the two fixed points shown in Fig. 5. We observe that the location of the maximum temperature in the contact zone changes during a transient contact. The temperature is given in Figs. 9(a) and 9(b) for two dimensionless contact times of 0.03 and 0.29, respectively. At the beginning of a contact, the location of the maximum temperature is in the middle of the contact zone as
shown in Fig. 9(a). During the contact, when the sphere penetrates into the flat, the location of the maximum interface temperature is shifted toward the trailing edge as shown in Fig. 9(b). We observe from Fig. 8 that the dimensionless temperature rise at the bottom of the sphere (dashed line) reaches its maximum value at $t/t_c = 0.28$. This value does not coincide with the maximum penetration of the sphere, which occurs at $t/t_c = 0.22$ (Fig. 4). This is due to the fact that the maximum temperature builds up close to the trailing edge of the contact zone as shown in Fig. 9(b) delaying the occurrence of the maximum temperature from the occurrence of the maximum penetration. After the maximum temperature is reached, the temperature decreases rapidly due to the reduced heat generation with the decrease in the contact pressure. Finally, Fig. 8 shows the rapid temperature rise of the fixed point on the moving flat (solid curve). The rapid temperature rise is generally described as “thermal flash.” It occurs at the dimensionless contact time of 0.21. After the highest temperature is reached, the temperature of the fixed point on the moving flat decays rapidly due to thermal conduction.

Figure 10 shows the dimensionless maximum temperature rise as a function of the dimensionless vertical initial velocity. We define $\Delta T_{max}^c$ as the modified critical maximum temperature rise given by Eq. (13). This value can be expressed as

$$\Delta T_{max}^c = \frac{V_{y}}{V_{y_c}}^{0.52}$$

(16)

and includes the effect of tangential velocity of the flat $V_x$. The best fit of the numerical results is shown by the solid curve in Fig. 10 and is given by

$$\Delta T_{max}^c = 5.8 \left( \frac{V_{y}}{V_{y_c}} \right)^{0.52}$$

(16)

We observe that the dimensionless maximum temperature rise increases monotonically with the dimensionless vertical initial velocity. This appears to be related to the increase in contact pressure, which leads to an increase in frictional heating. Clearly, the temperature rise is a function of the material properties, impact conditions, and sphere radius.

For impact conditions corresponding to a typical slider-disk contact, temperatures exceeding the Curie temperature of magnetic material can be obtained as was shown in Ref. [2]. The solution presented in Figs. 10–17 is independent of parameters such as material properties, impact conditions, and sphere radius for a wide range of parameters investigated ($8 < V_{y_c} < 400$, $0.04 < P_{e,max} < 10$, $0 < \mu < 0.3$, $0.3 < \nu < 0.33$, $35 < E/L_f < 220$, $0.07 < (A_{max}/\pi)^{1/3} < R < 0.4$). Therefore, Eq. (16) can be used, for example, to minimize the maximum temperature rise in a transient contact. This would be important in decreasing thermal erasure of magnetic information [2]. In order to minimize the maximum temperature rise, the critical vertical initial velocity (Eq. (9)) should be increased and the modified critical maximum temperature rise (Eq. (15)) should be decreased. Figure 11 shows the dimensionless maximum contact force as a function of the dimensionless vertical initial velocity for values of the friction coefficient of 0.0 and 0.3. The solid curve fitting the numerical results is given by
As expected, higher dimensionless vertical initial velocities cause larger dimensionless maximum contact force. We observe that the dimensionless maximum contact force is affected by the coefficient of friction for \( V_y / V_{y_c} > 50 \).

In Fig. 11, the dynamic Hertz solution [30] is also shown, indicated by the dashed line and assuming only elastic deformation. We observe that good correlation between the numerical results and the dynamic Hertz solution exists at dimensionless vertical initial velocities \( V_y / V_{y_c} < 50 \), i.e., when the amount of plastic deformation is small. However, if the dimensionless vertical initial velocity increases, plastic deformation in the soft flat leads to lower maximum contact force in comparison to a Hertzian contact.

Figure 12 shows the dimensionless maximum contact area as a function of the dimensionless vertical initial velocity for friction coefficients of 0.0 and 0.3, respectively. The solid curve fitting the numerical results is given by

\[
A_{\text{max}} / A_c = (1.15 + 0.6 \mu) \left( \frac{V_y}{V_{y_c}} \right)^{0.87} \tag{18}
\]

Due to “plowing,” the maximum contact area is not circular and appears to be a function of the dimensionless vertical initial velocity and the coefficient of friction. The dynamic Hertz solution [30] is shown by the dashed line in Fig. 12 for comparison. For light impacts, when the plastic deformation is insignificant, good agreement is observed with the dynamic Hertz solution. As the vertical initial velocity increases and the sphere penetrates deeper into the soft flat material, we observe that the dimensionless maximum contact area becomes increasingly larger than that predicted by the dynamic Hertz solution. This result is related to the fact that the elastic Hertzian contact theory does not consider material softening due to plastic deformation.

Figure 13 shows the dimensionless maximum penetration as a function of the dimensionless vertical initial velocity. The solid curve fitting the numerical results is given by

\[
\frac{\omega_{\text{max}}}{\omega_c} = 0.72 \left( \frac{V_y}{V_{y_c}} \right)^{0.9} \tag{19}
\]

We observe that the dimensionless maximum residual penetration is independent of the coefficient of friction. When the dimensionless vertical initial velocity is small, the maximum residual penetration is negligible due to elastic recovery of the flat. The dynamic Hertz solution [30] is indicated by the dashed line in Fig. 13. The difference between the two solutions stems from the plastic deformation of the flat leading to higher maximum penetration in comparison to the elastic Hertz solution.

Figure 14 shows the dimensionless maximum residual penetration as a function of the dimensionless vertical initial velocity. The solid curve fitting the numerical results is given by

\[
\frac{\omega_{\text{res, max}}}{\omega_c} = 0.2 \left( \frac{V_y}{V_{y_c}} \right)^{1.1} \tag{20}
\]

We observe that the dimensionless maximum residual penetration is independent of the coefficient of friction, as in the case of the dimensionless maximum penetration. As expected, more severe contact with higher dimensionless vertical initial velocity leads to increased plasticity in the contact and results in higher dimensionless maximum penetration. Equation (20) can be used to optimize the head-disk interface during slider-disk contacts by proper selection of material properties. The combination of Eqs. (19) and (20) is presented in Fig. 15 as dimensionless maximum residual penetration versus dimensionless maximum penetration, i.e.,

\[
\frac{\omega_{\text{max, res}}}{\omega_c} = 0.3 \left( \frac{\omega_{\text{max}}}{\omega_c} \right)^{1.22} \tag{21}
\]

We observe that with increasing severity of impacts the elastic recovery becomes negligible in comparison to plastic deformation.
and the slope of the solid curve approaches unity. In a multilayered disk structure such as used in hard disk drives, the maximum residual penetration of the recording layer must be minimized. This can be accomplished by increasing the critical interference $\omega_c$ of the disk material by increasing its yield strength, decreasing its modulus of elasticity or using a slider with larger corner radius representing a larger sphere radius.

The numerical solution of Etsion et al. [32] for quasi-static vertical loading and unloading of a spherical elastic-plastic frictionless contact is also shown in Fig. 15 for comparison. For light impacts, which correspond to small values of the dimensionless maximum penetration, good agreement is observed. However, with increasing dimensionless maximum penetration, the deviation between the two solutions increases. Clearly, for light impacts, for which the effect of tangential friction leading to plastic shearing of the flat surface is small, a quasi-static solution can be used. Otherwise, a fully transient solution is needed.

Figure 16 shows a comparison of the quasi-static solution of Kogut and Etsion [33] with the present solution for the dimensionless maximum contact force versus dimensionless maximum penetration. We observe good agreement between both solutions for small values of the dimensionless maximum penetration. However, with increasing impact severity, the present solution predicts higher values of the dimensionless maximum contact force.

Figure 17 shows the dimensionless maximum contact area as a function of the dimensionless maximum penetration for the present solution and the results of Kogut and Etsion [33]. We observe that good agreement exists at small values of $\omega_{\text{max}}/\omega_c$. However, as $\omega_{\text{max}}/\omega_c$ increases, the deviation between both solutions increases. This deviation appears to be related to the smaller contact area as a result of plowing shown in Figs. 5 and 9(a).

After completion of the present work, a new publication by Yu et al. [34] was brought to our attention treating a similar problem as the one in this paper. In particular, the oblique impact between a sphere and a layered disk was studied using finite element analysis. Results were obtained for maximum temperature rise and maximum penetration as a function of vertical initial velocity. The results in Ref. [34] are limited mainly to the elastic regime of deformations, because the calculated critical vertical velocity is $V_{y, c} = 0.059 \text{ m/s}$ and the maximum value of the vertical initial velocity used was $V_y = 0.06 \text{ m/s}$. Therefore, the value of $V_y/V_{y, c} = 1.02$, which is slightly above the elasticity limit of the contact. In the present paper much higher values of the dimensionless vertical initial velocities ($V_y/V_{y, c} > 8$) are investigated leading to excessive plastic deformations. Due to the different regimes of deformation studied in both papers (elastic in Ref. [34] and elastic-plastic in the present paper) only a qualitative comparison of the results of both papers can be made. Similar to Ref. [34], we have observed that for “light impacts” $V_y/V_{y, c} \approx 8$ the calculated results of the maximum temperature rise and maximum penetration are very close to the results predicted by the analytical elastic solutions of Refs. [8] and [30]. Comparing the results obtained in Ref. [34] with the present results, we observe a similar trend for
the maximum temperature rise and the maximum penetration as a function of vertical initial velocity, as expected.

5 Conclusion

The effect of impact parameters, material properties and sphere radius for a thermo-elastic-plastic transient contact between an impacting rigid sphere and a deformable moving flat was studied using finite element analysis. Nondimensional solutions were obtained for the maximum temperature rise, the maximum contact force, the maximum contact area, the maximum penetration, and the maximum residual penetration at the sphere-flat interface as a function of the vertical initial velocity of the sphere. We conclude:

1. The dimensionless maximum temperature rise, maximum contact force, maximum contact area, and maximum residual penetration are strong functions of the dimensionless vertical initial velocity.
2. For “light impacts” where plastic deformation and the effect of tangential velocity are small, elastic and quasi-static elastic-plastic solutions can be used to describe transient contacts.
3. The dimensionless solutions presented in this paper can be used, to optimize the slider-disk interface by reducing frictional heating and plastic deformation of the recording layer and, thereby, minimizing erasure of information in a hard disk drive.

Nomenclature

\( V_v \) = vertical initial velocity of sphere  
\( V_f \) = tangential velocity of flat  
\( R \) = sphere radius  
\( m \) = sphere mass  
\( E \) = equivalent modulus of elasticity  
\( Y \) = yield strength of flat  
\( \nu \) = Poisson ratio  
\( t \) = contact time  
\( \omega \) = penetration (interference)  
\( P \) = contact force  
\( A \) = contact area  
\( a \) = contact area radius  
\( p \) = mean contact pressure  
\( \Delta T \) = temperature rise  
\( \mu \) = coefficient of friction (sliding)  
\( k \) = thermal conductivity  
\( c \) = specific heat  
\( \rho \) = material density  
\( Pe \) = Peclet number  
\( K \) = thermal diffusivity

Subscripts

\( s \) = sphere  
\( f \) = flat  
\( c \) = critical  
\( \text{max} \) = maximum  
\( \text{tot} \) = total  
\( \text{res} \) = residual  
\( TK \) = Tian and Kennedy

Appendix

A.1 Theoretical Background

The transient contact between the sphere and the flat is solved using the finite element software package LS-DYNA [28].

The momentum equation in the updated Lagrangian formulation is the basis of the finite element method [28]. The conservation of momentum for a continuous body is given by

\[
\sigma_{ij} + \rho b_i = \rho \ddot{u}_i
\]  \hspace{1cm} (A1)

where \( \sigma_{ij} \) is the Cauchy stress tensor, while \( \rho, b_i, \) and \( \ddot{u}_i \) are mass density, body force per volume, and acceleration, respectively. The weak form (principle of virtual work) of the momentum Eq. (A1), taking into account body force density and surface traction (or contact force) \( t_i \), is given by

\[
\int_V \rho \ddot{u}_i \delta u_i dV + \int_V \sigma_{ij} \delta u_j dV = \int_V \rho b_i \delta u_i dV + \int_S t_i \delta u_i dS
\]  \hspace{1cm} (A2)

where \( \delta u_i \) is the nodal velocity. Equation (A2) can be rewritten in the matrix form

\[
M \ddot{u} + Ku = F
\]  \hspace{1cm} (A3)

where \( M \) is the lumped mass matrix, \( \ddot{u} \) is the nodal acceleration vector, \( K \) is the stiffness matrix, \( u \) is the nodal displacement vector, and \( F \) is the body force vector. In LS-DYNA Eq. (A3) is solved using explicit time integration with central differences [28]. The solution at step \( n+1 \) is calculated from the known solution at the previous time step \( n \) given by

\[
u^{n+1} = \left( \frac{M}{\Delta t^2} \right)^{-1} \left[ F - Ku^n + \frac{M}{\Delta t^2} (2u^n - u^{n-1}) \right]
\]  \hspace{1cm} (A4)

where \( \Delta t \) is the time increment.

The thermal equilibrium can be expressed as

\[
pcT - k \dot{T}_{,i} - q = 0
\]  \hspace{1cm} (A5)

where \( c \) is the specific heat, \( T \) is the temperature, \( k \) is the thermal conductivity, matrix, \( \dot{T} \) is the temperature change with time, and \( q \) is the heat generation per unit volume and time. The weak form of the principle of conservation of energy, when applied to the divergence theorem, gives

\[
\int_V p c \dot{T} \delta \dot{T} dV + \int_V k T_{,i} \delta \dot{T}_i dV = \int_V q \delta \dot{T} dV - \int_S \Psi \delta \dot{T} dS
\]  \hspace{1cm} (A6)

where \( \Psi \) is the heat flux, directed outward in space and normal to the boundary surface. Equation (A6) can be rewritten in matrix form as

\[
C_T \dot{T} + K_T T = Q
\]  \hspace{1cm} (A7)

where \( C_T \) is the heat capacitance matrix, \( \dot{T} \) is the change of nodal temperature with time, \( K_T \) is the thermal conductivity matrix, \( T \) is the nodal temperature vector, and \( Q \) is the thermal force vector including heat generation due to frictional heating. In LS-DYNA, Eq. (A7) is solved using a backward integration scheme [28].

In coupled thermomechanical analysis, mechanical equilibrium as well as thermal equilibrium must be satisfied. Therefore, Eqs. (A3) and (A7) are combined as follows

\[
\begin{bmatrix}
M & 0 \\
0 & 0 \\
0 & C_T
\end{bmatrix}
\begin{bmatrix}
\ddot{u} \\
\dot{T}
\end{bmatrix} +
\begin{bmatrix}
0 & 0 \\
0 & C_T
\end{bmatrix}
\begin{bmatrix}
\dot{u} \\
\dot{T}
\end{bmatrix} +
\begin{bmatrix}
K & 0 \\
0 & K_T
\end{bmatrix}
\begin{bmatrix}
\ddot{u} \\
\dot{T}
\end{bmatrix} =
\begin{bmatrix}
F \\
Q
\end{bmatrix}
\]  \hspace{1cm} (A8)

The heat energy generated during contact is assumed to be equal to the energy dissipated by the frictional sliding energy \( q_s \) per unit area and time

\[
q_s = \mu \rho \dot{v}
\]  \hspace{1cm} (A9)

where \( \mu \) is the friction coefficient, \( p \) is the contact pressure, and \( \dot{v} \) is the relative sliding velocity between the two contacting bodies. The division of thermal energy between the sphere and the flat is discussed in Sec. 2.3.
In closing, we would like to mention that elastic wave propagation phenomena during the contact between the sphere and the flat were not observed in the present model, since those phenomena are on a time scale that is orders of magnitude shorter than that of a typical sphere-flat contact for the range of parameters studied in this paper.

A.2 Details of Finite Element Model. The three-dimensional mesh used in this study consists of 16,780 four-node tetrahedral and 23,280 eight-node hexahedron constant stress elements with one point of integration [28,35] for the sphere and the flat, respectively. Special surface contact elements are used (contact_automatic_surface_to_surface_thermal [35]); 1122 contact elements for the sphere and 3000 contact elements for the flat. A total of 30,636 nodes is used. The sphere and the flat were gradually meshed with increasingly finer mesh near the contact zone as shown in Fig. 2. Typical calculation times were on the order of 2 h on a HP computer with four central processing units of 2.33 GHz and 8 Gb RAM.

A.3 Finite Element Model Validation. The thermomechanical finite element model was verified in several ways. At first, mesh convergence was checked to find optimal and efficient mesh size parameters. The convergence of the mechanical and thermal time scale was also tested at every stage of the mesh convergence.

Second, the mechanical and thermal results of the finite element model were compared with analytical predictions. To test the mechanical solution, the heat generation from sliding was switched off, and the tangential velocity was set equal to zero. The maximum interference (penetration), contact load, contact area, and contact time were compared with the dynamic Hertz solution [30] for different values of the vertical initial velocity of a sphere. The difference between numerical and analytical results was found to be less than 3% in the elastic regime of deformation assuming a frictionless normal contact.

For the thermal solution, a constant normal load was applied to the sphere (rather than a vertical initial velocity) in the elastic regime of deformation. Then, a constant tangential velocity was applied to the flat, and the steady-state maximum surface temperature rise was compared with the analytical solution for maximum temperature rise given in Ref. [8]. The difference between numerical results and analytical predictions was found to be less than 1% for the maximum surface temperature rise for the range of Peclet numbers investigated (0.1 ≤ Pe ≤ 5).

Finally, a so-called mass-scaling technique [36] was implemented in order to decrease computation time. The original results of maximum temperature rise, equivalent von Mises stress and strain, penetration (interference), and contact time without mass-scaling were compared with values obtained using mass-scaling for typical cases of impact conditions. The mass-scaling parameters were set to yield an error of less than 1% in both cases.

References


