A method and apparatus for multicasting of a multi-packet message are disclosed. Data to be transmitted as a message are divided into N sets, each set being encoded to generate encoded data. A set of parity bits is separated from each of the N sets of encoded data. The N sets of separated parity bits are encoded by a systematic code with a predetermined distance S across the N sets, resulting in N parity-bit packets. The N parity-bit packets are encoded with a code that is selected so that each receiving station decodes the N parity-bit packets with a high probability. The N-packet message, comprising the N sets of encoded data less the separated bits, and the N packets are multicasted. If less than S packets of the N-packet message fail to decode at a receiving station, the receiving station recovers all N packets using the N packets.
CALCULATE PUNCTURED BYTES $R_2$ FOR EACH CORRECT RECEIVED PACKET

DECODE THE $N'R_2$ PARITY BYTES

CALCULATE THE PUNCTURED BYTES $R_2$ FOR THE ERASED PACKETS

DECODE ALL ERASED PACKETS

FIG. 3
DECODE ALL PARITY BYTES OF THE R-S CODE

\( i = 1, 1, R_2 \)

RECOVER ALL PUNCTURED PARITY BYTES AT THE i-th PARITY BYTE POSITION

DECODE ALL PACKETS THAT CAN BE DECODED WITH KNOWLEDGE OF PARITY BYTE AT THE i-th POSITION

COMPUTE ALL MISSING PARITY BYTES FOR THE DECODED PACKET

FIG. 4
METHOD AND AN APPARATUS FOR USE OF CODES IN MULTICAST TRANSMISSION

BACKGROUND OF THE INVENTION

[0001] 1. Field of the Invention

[0002] The current invention relates to the field of communications. More particularly, the present invention relates to the use of codes in multicast transmission.

[0003] 2. Description of the Related Art

[0004] Communication systems have been developed to allow transmission of an information signal from an origination station to one or more physically distinct destination stations. In transmitting the information signal from the origination station over a communication channel, the information signal is first converted into a form suitable for efficient transmission over the communication channel. As used herein, the communication channel comprises a single path over which a signal is transmitted. Conversion, or modulation, of the information signal involves varying a parameter of a carrier wave in accordance with the information signal in such a way that the spectrum of the resulting modulated carrier is confined within the communication channel bandwidth. At the destination station the original information signal is replicated from the modulated carrier wave received over the communication channel. Such a replication is generally achieved by using an inverse of the modulation process employed by the origination station.

[0005] Modulation also facilitates multiple-access, i.e., simultaneous transmission and/or reception of several signals over a common communication channel. Multiple-access communication systems often include a plurality of remote subscriber units requiring intermittent service of relatively short duration rather than continuous access to the common communication channel. Several multiple-access techniques are known in the art, such as Time Division Multiple-Access (TDMA), Frequency Division Multiple-Access (FDMA), and Amplitude Modulation (AM). Another type of multiple-access technique is used in a Code Division Multiple-Access (CDMA) spread spectrum system that conforms to the "TIA/EIA/IS-95 Mobile Station-Base Station Compatibility Standard for Dual-Mode Wide-Band Spread Spectrum Cellular System," hereinafter referred to as the IS-95 standard. The use of CDMA techniques in a multiple-access communication system is disclosed in U.S. Pat. No. 4,901,207, entitled "SPREAD SPECTRUM MULTIPLE-ACCESS COMMUNICATION SYSTEM USING SATELLITE OR TERRESTRIAL REPEATERS," and U.S. Pat. No. 5,103,459, entitled "SYSTEM AND METHOD FOR GENERATING WAVEFORMS IN A CDMA CELLULAR TELEPHONE SYSTEM," both assigned to the assignee of the present invention and incorporated herein by reference.

[0006] A multiple-access communication system may carry voice and/or data. An example of a communication system carrying both voice and data is a system in accordance with the IS-95 standard, which specifies transmitting voice and data over the communication channel. A method for transmitting data in code channel frames of fixed size is described in detail in U.S. Pat. No. 5,504,773, entitled "METHOD AND APPARATUS FOR THE FORMATTING OF DATA FOR TRANSMISSION," assigned to the assignee of the present invention. In accordance with the IS-95 standard, the data or voice is partitioned into code channel frames that are 20 milliseconds wide with data rates as high as 14.4 kbps. Additional examples of communication systems carrying both voice and data are communication systems conforming to the "3rd Generation Partnership Project" (3GPP), embodied in a set of documents including Document Nos. 3G TS 25.211, 3G TS 25.212, 3G TS 25.213, and 3G TS 25.214 (the W-CDMA standard), or "TR-45.5 Physical Layer Standard for cdma2000 Spread Spectrum Systems" (the IS-2000 standard).

[0007] An example of a data only communication system is a high data rate (HDR) communication system, such as the communication system disclosed in co-pending application Ser. No. 08/963,386, entitled "METHOD AND APPARATUS FOR HIGH RATE PACKET DATA TRANSMISSION," filed Nov. 3, 1997, assigned to the assignee of the present invention. The HDR communication system defines a set of data rates, ranging from 38.4 kbps to 2.4 Mbps, at which an origination terminal (Access Point, AP) may send data to a receiving terminal (Access Terminal, AT).

[0008] The information signal to be exchanged among the terminals in a communication system is often organized into a plurality of packets. For the purposes of this description, a packet is a group of bytes, including data (payload) and control elements, arranged into a specific format. The control elements comprise, e.g., a preamble and a quality metric. The quality metric comprises, e.g., Cyclic Redundancy Check (CRC), parity bit(s), and other types of metric known to one skilled in the art. The packets are usually formatted into a message in accordance with a communication channel structure. The message, appropriately modulated, traveling between the origination terminal and the destination terminal is affected by characteristics of the communication channel, e.g., signal-to-noise ratio, fading, time variance, and other such characteristics. Such characteristics affect the modulated signal differently in different communication channels. Consequently, transmission of a modulated signal over a wireless communication channel requires different considerations than transmission of a modulated signal over a wire-like communication channel, e.g., a coaxial cable or an optical cable. In addition to selecting modulation appropriate for a particular communication channel, other methods for protecting the information signal have been devised. Such methods comprise, e.g., encoding, symbol repetition, interleaving, and other methods known to one of ordinary skill in the art. However, these methods increase overhead. Therefore, an engineering compromise between reliability of message delivery and the amount of overhead must be made. Even with the above-discussed protection of information, the conditions of the communication channel can degrade to the point at which the destination station possibly cannot decode (erases) some of the packets comprising the message. In data-only communications systems, the cure is to re-transmit the non-decoded packets using an Automatic Retransmission reQuest (ARQ) made by the destination station to the origination station.

[0009] Often a message is multicast transmitted by the origination station. For the purposes of this document, multicast transmission means transmission of a message that is intended to be received by a plurality of destination stations. However, for the above-discussed reasons, the destination stations may fail to decode a subset of the
message. Furthermore, subsets of packets that are not decoded may differ from one destination station to another destination station.

[0010] Based on the foregoing, there is a need in the art for a method and an apparatus for multicast transmission that allows each destination station to decode the destination station’s faulty subset of packets, and to reconstruct the multicast message.

SUMMARY OF THE INVENTION

[0011] The present invention is directed to a method and an apparatus allowing each destination station to decode a multcasted message from an origination station. In one aspect of the invention, the origination station processes each of a plurality of data sets to generate a processed data set and a parity block for each data set, processes a plurality of the parity blocks to generate at least one packet; and transmits the plurality of processed data sets and the at least one packet. The destination station receives a plurality of packets comprising the message, and at least one other packet, decodes each packet of the plurality of packets comprising the message; and decodes each incorrectly decoded packet in accordance with the at least one other packet when a number of the incorrectly decoded packets is less than or equal to a pre-determined code distance.

[0012] In another aspect of the invention, the origination station processes each of a plurality of data sets to generate a processed data set and a parity block for each data set, transmits the plurality of processed data sets as packets, receives signals containing information about incorrectly decoded packets, and when the signals are received processes a plurality of the parity blocks to generate at least one packet; and transmits the at least one packet. The destination station decodes each packet of the received plurality of packets comprising the message; transmits a report containing at least one number; receives at least one packet in response to the report; and decodes the incorrectly decoded packets in accordance with the received at least one packet.

BRIEF DESCRIPTION OF THE DRAWINGS

[0013] FIGS. 1A-1B illustrate a block diagram of a communication system with shared redundancy;

[0014] FIGS. 2A-2D illustrate a block diagram of a communication system with shared redundancy with punctured bytes;

[0015] FIG. 3 illustrates a method used by a j-th destination station to recover erased packets; and

[0016] FIG. 4 illustrates another method used by the j-th destination station to recover erased packets.

DETAILED DESCRIPTION OF PREFERRED EMBODIMENTS

[0017] In order to compare performance of the methods and apparatus in accordance with different embodiments of the present invention, a concept of a baseline system is introduced.

[0018] Baseline System

[0019] In a baseline system, each of N packets, comprising a message, is sent as an element of a code C, i.e., each packet is encoded by a code C. The code C contains an amount of redundancy designed to satisfy message delivery with a probability P_{C, Average} under certain, i.e., average, conditions of a communication channel. Furthermore, the code C also has the property that the maximum number of packets organized into an N packet message that cannot be decoded correctly by a destination station is less than or equal to S with a probability P_{C}. Consequently, when the variation in the condition of the communication channel between the origination station and the particular destination station(s) degrades below the design criteria, a subset of the packets comprising the message fails to be decoded at the destination station(s).

[0020] The amount of redundancy necessary to satisfy reliable message delivery under certain conditions of the communication channel is the only design requirement for selection of the code C. Thus, the code C may be, e.g., an algebraic block code such as a Reed-Solomon code, a state machine code such as a convolutional or trellis code, a classical concatenated code, a serial/parallel concatenated turbo code, or a binary convolutional code, as well as other codes known to one skilled in the art.

[0021] For the purposes of quantitatively comparing various methods used by a destination station to decode the packets comprising the message, it can be assumed without loss of generalization, that an input signal comprises N blocks of K bytes, which result in (K+R,) coded bytes when encoded with the code C. (The term byte as used in this document includes a 1-bit byte.) R_1 denotes number of additional byte, related to the number of the input signal bytes K. R_1 is the measure of the code C redundancy. Therefore, in the baseline system, each of the N packets is transmitted as a sequence of (K+R,) bytes.

[0022] Automatic Retransmission ReQuest

[0023] To improve the performance of the baseline system, the non-decoded packets of the message may be re-transmitted using a modification of an ARQ. In such an ARQ arrangement, every destination station reports to the origination station a list of packets that were not decoded correctly. The origination station re-transmits every packet that appears on any of the lists. The process of reporting and re-transmitting is repeated until every destination station decodes all the packets of the message. The fractional overhead (FO) of the ARQ system is analyzed as follows. Let P denote the probability that at least one destination station fails to decode a packet sent as an element of the code C. The average number of bytes (ANB) to be re-transmitted in order to deliver the message to all the destination stations is given by the following equation:

\[ ANB \geq \frac{N \cdot (K + R_1)}{(1 - P)} \]
Therefore, the fractional overhead relative to the baseline system (FO_{ARQ}) is:

\[ FO_{ARQ} = \frac{P_e}{1 - P_e} \]  \hspace{1cm} (2)

Because the probability \((1-P_e)\) decreases exponentially with an increase in the number of destination stations, the ARQ arrangement efficiency decreases with increasing number of the destination stations. Furthermore, a variable latency exists in this ARQ arrangement.

[0026] Full Redundancy per Packet

Another approach allowing every destination station to decode all the packets in the message is to design the communication system in accordance with the worst-case condition of the communication channel. In such an approach, each packet is encoded by a code \(C_3\). The code \(C_3\) is selected so that each encoded packet is decoded correctly with a probability \(P_{C3}\) under the worst-case condition of the communication channel by every destination station. A relationship between a required probability of receiving the message \(P_{message}\) under the worst-case condition of the communication channel and the probability \(P_{C3}\) is given by the equation:

\[ P_{message} = N \cdot P_{C3} \]  \hspace{1cm} (3)

The assumption of the worst-case condition of the communication channel for each transmitted packet makes this scheme very inefficient. A measure of the inefficiency is the number of extra bytes transmitted as compared to the baseline system. For the purposes of quantitatively characterizing this method it can be assumed, that \(K\) bytes of an input signal result in \((K+R)\) coded bytes when encoded with the code \(C_3\). One of ordinary skill in the art recognizes that because the code \(C_3\) is designed to deliver the message under the worst-case condition of the communication channel while the code \(C_1\) was designed to deliver the message under average conditions of the communication channel, the amount of redundancy of the code \(C_3\) is greater than the amount of redundancy of the code \(C_1\). Consequently, the number of parity bytes \(R_3\) is greater than the number of parity bytes \(R_1\). This relationship is expressed as:

\[ R_3 > R_1 > R_{13} \]  \hspace{1cm} (4)

where \(R_{13}\) is the number of extra parity bytes of the code \(C_3\) in relation to the code \(C_1\). Because \(R_2\) extra bytes are transmitted per packet in this arrangement relative to the baseline system, the \(N\) packets comprising the message contain a total of \(N\cdot R_2\) extra bytes relative to the baseline system. The fractional overhead (FOFRPP) is:

\[ FO_{FRPP} = \frac{R_{13}}{K + R_1} \]  \hspace{1cm} (5)

The amount of redundancy necessary to satisfy message delivery under the worst-case condition of the communication channel is the only design requirement on the code \(C_3\). Thus, the code \(C_3\) may be, e.g., an algebraic block code such as a Reed Solomon code, or a state machine code such as a convolutional or trellis code. The code \(C_3\) may also be a classical concatenated code, a serial/parallel concatenated turbo code, or a binary convolutional code, as well as other codes known to one skilled in the art.

[0031] Shared Redundancy

[0032] FIG. 1A illustrates a conceptual block diagram of a communication system employing shared redundancy.

[0033] In one embodiment, a Data Source (DS) 102 generates an information signal to be multicast. The information signal is divided into \(N\) blocks, each block comprising \(K\) bytes. The \(N\) blocks are provided to a first encoder (EC1) 104, which encodes each of the \(N\) blocks with a code \(C_1\), providing a packet comprising \(K+R_1\) code bytes as shown in FIG. 1A. The code \(C_1\) is selected so that the maximum number of packets organized into an \(N\) packet message that cannot be decoded correctly by a destination station is less than or equal to \(S\) with probability \(P_{C1}\).

[0034] The \(N\) blocks are also provided to a second encoder (EC2) 106. Referring to FIG. 1B, in one embodiment, the \(i\)-th byte of each of the \(N\) blocks is combined to form an \(i\)-th input data block 120. Each input data block 120 is then provided to the encoder 106, which is a systematic block encoder 106 in one embodiment. For the purposes of this document, a systematic code comprises a permutation of information (systematic) bytes and parity bytes. Thus, a systematic code is defined by the following equation:

\[ x = s(u,p) \]  \hspace{1cm} (6)

where:

\[ \begin{align*}
  &x \text{ is the coded signal;} \\
  &u \text{ is a permutation;} \\
  &p \text{ are parity bytes.}
\end{align*} \]

[0040] The systematic block encoder 106 encodes each input data block 120 with a systematic code \(C_s\), having a minimum distance greater than \(S\), resulting in \(K\) encoded packets 122. Each encoded packet 122 comprises \(N\) systematic bytes and \(N\) parity bytes. In one embodiment, a systematic Reed-Solomon (RS) code with \(N\times S\), which is guaranteed to have a minimum distance of \((S+1)\), is used.

[0041] In another embodiment, the NK systematic bytes of the \(N\) packets are provided to a systematic block encoder 106. The systematic block encoder 106 encodes the NK systematic bytes with a systematic code \(C_s\). The systematic code \(C_s\) is selected to be capable of correcting segments in the NK systematic bytes when each of the segments contains a maximum of \(S\) errors. A segment comprises the \(i\)-th systematic byte of each of the \(N\) packets. In one embodiment, a Reed-Solomon code having a minimum distance greater than \(K\times S\) is used.

[0042] The encoding by the encoder 106 results in \(N\) systematic bytes and \(N\) parity bytes. The \(N\) parity bytes computed by the encoder 106 are provided to a third encoder (EC3) 108. The encoder 108 encodes the \(N\) parity bytes with the code \(C_3\). The code \(C_3\) is selected so that a packet sent as an element of the code \(C_3\) is decoded correctly with a probability \(P_{C3}\) by every destination station. A relationship between a required probability of receiving the message \(P_{message}\), the probability \(P_{C1}\), and the probability \(P_{C3}\) is given by the following equation:

\[ P_{message} \approx P_{C1} \cdot P_{C3} \]  \hspace{1cm} (7)
Therefore, given a reliability requirement for message delivery expressed in terms of $P_{\text{message}}$, Equation (7) is used for selection of the codes $C_2$ and $C_3$.

The purpose of encoding the Nt parity bytes by the code $C_2$ is to deliver the encoded Nt parity bytes 122 with reliability expressed in terms of $P_{C_3}$. Consequently, there is no restriction on the organization of the encoded Nt parity bytes 122. Therefore, the encoded Nt parity bytes 122 may be organized in an arbitrary number of packets. Consequently, in one embodiment, the encoded Nt parity bytes 122 form one packet. In another embodiment, the encoded Nt parity bytes 122 are organized into a plurality of packets.

Referring back to FIG. 1A, the output signals of the encoder 104 and the encoder 108 are provided to a transmitter (TX) 110. The transmitter 110 performs processing of the provided signals in accordance with a modulation scheme used. In one embodiment, the modulation is carried out in accordance with the requirements of a wireless communication channel. The transmitter 110 then transmits the N packets provided by the encoder 104 and the packet or packets provided by the encoder 108 over a communication channel 112. Although a wireless communication channel is shown in FIG. 1, one of ordinary skill in the art recognizes that the communication channel 112 can be wire-like, e.g., a coaxial cable, an optical cable, etc. In such an embodiment, the modulation is carried out in accordance with the requirements of the particular wire-like communication channel.

A receiver (RX) 114 receives the N-packet message and the packet or packets of the encoded Nt parity bytes 122 of FIG. 1B. The receiver 114 processes the packets in accordance with a demodulation scheme. Generally, an inverse of the modulation process employed by the transmitter 110 is used. The processed packets are provided to a decoder (DC) 116. The decoder 116 decodes each of the N received message packets. As discussed, at most S of the N packets fail to decode. The decoder 116 then decodes the Nt parity bytes and uses the Nt parity bytes to reconstruct the non-decoded packets. The reconstruction may be carried out in accordance with any method known to one of ordinary skill in the art. For example, a method for a systematic Reed Solomon (RS) code is disclosed in Truong, T.-K., Jeng, J., H., and Hung, K.-CH., Inversionless Decoding of Both Errors and Erasures of Reed-Solomon Code. The decoded packets are provided to a data sink (DSK) 118.

The Total Number of Bytes (TNB) transmitted by this scheme when RS is used is:

$$ TNB = N(K + R_t) + N(K + R_s) $$(8)

Because $N(K + R_t) = S(K + R_s)$ extra bytes are transmitted per packet relative to the baseline system, the fractional overhead (FOSR) is:

$$ FOSR = \left( \frac{S}{N} \right) \left( 1 + \frac{R_s}{K + R_s} \right) $$

$$ \frac{R_s}{K + R_s} $$

Shared Redundancy with Punctured Bytes

FIG. 2A illustrates a conceptual block diagram of a communication system employing shared redundancy with punctured bytes.

In one embodiment, a Data Source (DS) 202 generates an information signal to be multicast. The information signal is divided into N blocks, each block comprising K bytes, and the blocks are provided to an encoder (EC2) 204. The encoder 204 encodes each of the N blocks with a code $C_2$, providing a packet comprising $K + R_s$ code bytes as shown in FIG. 2B. The code $C_2$ is selected so that the maximum number of packets organized into an N-packet message that cannot be decoded correctly by a destination station is less than or equal to $S$.

The encoder 204 also computes $R_s$ parity bytes of a code $C_3$ for each of the N blocks, providing a packet structure illustrated in FIG. 2C. The packet structure encoded by the code $C_3$ comprises $K + R_s$ code bytes (i.e., a structure of a packet encoded by the code $C_3$) appended by the $R_s$ parity bytes. Consequently, the code $C_2$ is an extension of the code $C_3$. In other words, the code $C_3$ is a punctured version of the code $C_2$. The code $C_3$ is selected so that a packet sent as an element of the code $C_3$ is decoded correctly with a probability $P_{C_3}$ by every destination station.

One of ordinary skill in the art recognizes that different apparatuses and methods accomplish encoding of the information signal to provide the packet structures shown in FIG. 2B and 2C. However, as long as the particular apparatus and method provide the packet structures of FIG. 2B and 2C, a selection of a particular apparatus and method is a matter of implementation.

The NR3 parity bytes computed by the encoder 204 for the N-packet message are further encoded. In one embodiment, illustrated in FIG. 2D, the i-th parity byte of each of the N packets is combined to form an i-th input data block 220. Each input data block 220 is provided to a systematic block encoder (EC3) 206. The systematic block encoder 206 encodes each input data block 220 by a systematic code $C_3$, having a minimum distance greater than $S$, resulting in $R_s$ encoded packets. Each encoded packet comprises N systematic bytes and N parity bytes. In one embodiment, a systematic Reed-Solomon code with $N = S$, which is guaranteed to have a minimum distance of $(S + 1)$, is used. Although FIG. 2D illustrates the input data bytes 220 to be encoded in parallel, one of ordinary skill in the art recognizes that such an illustration is for pedagogical reasons only, and other arrangements, e.g., serial encoding, are possible.

In another embodiment (not shown), the NR3 parity bytes are provided to the systematic block encoder 206. The systematic block encoder 206 encodes the NR3 parity bytes with a systematic code $C_3$. The systematic code $C_3$ is selected to be capable of correcting segments in the NR3 parity bytes when each of the segments contains a maximum of $S$ errors. A segment comprises the i-th systematic byte of each of the N packets. In one embodiment, a systematic Reed-Solomon code having a minimum distance greater than $R_s S$ is used.

The above-described encoding results in NR3 systematic bytes and NR3 parity bytes. The NR3 parity bytes computed by the encoder 206 are provided to an encoder (EC4) 208. The encoder 208 encodes the NR3 parity bytes with the code $C_4$. The code $C_4$ is selected so that a packet sent as an element of the code $C_4$ is decoded correctly with a probability $P_{C_4}$ by every destination station. A relationship
between a required probability of receiving the message \(P_{\text{message}}\), the probability \(P_{\text{C2}}\), and the probability \(P_{\text{C3}}\) is given by the following equation:

\[
P_{\text{message}} = P_{\text{C2}} \cdot P_{\text{C3}}
\]

(10)

[0057] Therefore, given a reliability requirement for message delivery expressed in terms of \(P_{\text{message}}\), Equation (10) is used for selection of the codes \(C_2\) and \(C_3\).

[0058] The purpose of encoding the \(NR_2\) parity bytes with the code \(C_2\) is to deliver the encoded \(NR_2\) parity bytes \(224\) from an origination station to a destination station with reliability expressed in terms of \(P_{\text{C2}}\). Consequently, there is no restriction on the organization of the encoded \(NR_2\) parity bytes \(224\). Therefore, the encoded \(NR_2\) parity bytes \(224\) may be organized in an arbitrary number of packets. Consequently, in one embodiment, the encoded \(NR_2\) parity bytes \(224\) form one packet. In another embodiment, the encoded \(NR_2\) parity bytes \(224\) are organized into a plurality of packets.

[0059] Referring back to FIG. 2A, the output signals of the encoder 204 and the encoder 208 are provided to a transmitter 210. The transmitter 210 performs processing of the provided signals in accordance with the modulation scheme used. In one embodiment, the modulation is carried out in accordance with the requirements of a wireless communication channel. The transmitter 210 then transmits the \(N\) packets provided by the encoder 204 and the packet or packets provided by the encoder 208 over a communication channel 112. Although a wireless communication channel is shown in FIG. 2A, one of ordinary skill in the art recognizes that the communication channel 112 can be wire-like, e.g., a coaxial cable, an optical cable, etc. In such an embodiment, the modulation is carried out in accordance with the requirements of the particular wire-like communication channel.

[0060] A receiver 214 receives the \(N\)-packet message and the packet or packets of the encoded \(NR_2\) parity bytes \(224\) of FIG. 2D. The receiver 214 processes the packets in accordance with a demodulation scheme. Generally, an inverse of the modulation process employed by the transmitter 210 is used. The processed packets are provided to a decoder 216. The decoder 216 decodes each of the \(N\) received message packets. As discussed, at most \(S\) of the \(N\) packets fail to decode. The decoder 216 then decodes the \(NR_2\) parity bytes, and uses the \(NR_2\) parity bytes to recover the non-decoded packets using the method illustrated in FIG. 3. The decoded packets are provided to a data sink 218.

[0061] Referring to FIG. 3, in step 302, a method computes the \(R_2\) punctured bytes for each of the \(N\) message packets that were correctly decoded. (Thus, the decoder computes at least \((N-S)R_2\) punctured bytes in this manner.) The \((N-S)R_2\) punctured bytes are also the systematic bytes of the Reed-Solomon code. The method then continues in step 304.

[0062] In step 304, the packet or packets of the encoded \(NR_2\) parity bytes are decoded. Because the packet or packets were encoded by the code \(C_2\), decoding is successful with high reliability. The method then continues in step 306.

[0063] In step 306, the remaining \(SR_2\) punctured bytes, which are also the systematic bytes of the Reed-Solomon code, are recovered using the \(NR_2\) parity bytes, using the erasure correction capability of the Reed-Solomon code. An example of the erasure correction capability of the Reed-Solomon code is disclosed in Truong, T.-K., Jeng, J.-H., and Hung, C.-CH., Inversionless Decoding of Both Errors and Erasures of Reed-Solomon Code. The method then continues in step 308.

[0064] In step 308, the punctured bytes recovered in step 306 now provide enough redundancy to decode all of the packets that were not yet decoded.

[0065] Because the total number of extra bytes transmitted by this scheme relative to the baseline system is \([SR_2(1 + (R_1 + R_{11})/K)]\), the fractional overhead (FOSRPB) relative to the baseline scheme is given as:

\[
F_{OSRPB} = \frac{F_{OSRPB}}{F_{OS}} = \frac{R_2}{K}
\]

(11)

[0066] Comparing the results for the disclosed system with the Shared Redundancy System, yields:

\[
F_{OSRPB} = \frac{R_2}{K}
\]

(12)

[0067] Therefore, for \(R_2 << K\) the Shared Redundancy System with Punctured Bytes is more efficient than the Shared Redundancy System.

[0068] Table 1 shows the fractional overhead associated with the above-discussed methods for a typical set of parameter values.

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARQ</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>FO</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>FO for values:</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

[0069] Shared Redundancy with Punctured Bytes and ARQ

[0070] In another embodiment, referring back to FIGS. 2A-D, the information signal processing by the data source 202, the encoder 204, and the encoder 208 may be identical to the processing described above in the “Shared Redundancy with Punctured Bytes” embodiment.

[0071] The output signal of the encoder 204 is provided to a transmitter 210. The transmitter 210 performs processing of the provided signal in accordance with the processing described above in the “Shared Redundancy with Punctured Bytes” embodiment.
The receiver 214 at each destination station receives the N message packets and processes the packets in accordance with the demodulation scheme. Such processing is generally achieved by using an inverse of the modulation process employed by the transmitter 210. The processed packets are provided to a decoder 216. The decoder 216 decodes the N packets, and determines how many packets failed to decode. Each destination station then informs the origination station about how many packets the destination station was unable to decode. Let \( S_d \) denote the number of packets erased by the j-th destination station. Then, it is sufficient that the origination station send SR2 parity bytes of the Reed-Solomon code, where S' is given by Equation (13):

\[
S' = \max(S_d) \tag{13}
\]

The SR2 parity bytes are provided to an encoder 206. The processing of the SR2 parity bytes by the encoder 206 and the encoder 208 may be identical to the processing described above in the “Shared Redundancy with Punctured Bytes” embodiment.

The output signal of the encoder 208 is provided to a transmitter 210, which transmits the properly modulated signal to the receiver 214. The receiver 214 processes the packet or packets in accordance with a demodulation scheme, and provides the demodulated packet or packets to the decoder 216. The decoder 216 then uses the SR2 parity bytes to recover the non-decoded packets using the method illustrated in FIG. 3. The decoded packets are provided to the data sink 218.

If S' is considerably lower than S most of the time, then this embodiment is more efficient than the above-described “Shared Redundancy with Punctured Bytes” embodiment.

Modified Shared Redundancy with Punctured Bytes and ARQ

In another embodiment, the destination stations are configured to determine the number of first punctured bytes necessary to decode a packet that failed to decode when sent as an element of code C4. In accordance with the embodiment, referring to FIGS. 2A-2D, the information signal processing by the data source 202 and the encoder 204 may be identical to the processing described above in the “Shared Redundancy with Punctured Bytes” embodiment.

The NR2 parity bytes computed by the encoder 204 are further encoded. Referring to FIG. 2D, the i-th parity byte of each of the N packets is combined to form an i-th input data block 220. Each input data block 220 is provided to a systematic block encoder 206. The systematic block encoder 206 encodes each input data block 220 by a systematic code C4, having a minimum distance greater than S, resulting in an R2 encoded packets 222, each packet 222 comprising N systematic bytes and N' parity bytes. In one embodiment, a systematic Reed-Solomon (RS) code with N'-S, which is guaranteed to have a minimum distance of (S+1), is used. The above-described encoding results in NR2 systematic bytes and NR2 parity bytes.

The output signal of the encoder 204 of FIG. 2A is provided to a transmitter 210. The transmitter 210 performs processing of the provided signal in accordance with the processing described above in the “Shared Redundancy with Punctured Bytes” embodiment.

The receiver 214 at each destination station receives the N packets and processes the packets in accordance with the demodulation scheme. Such processing is generally achieved by using an inverse of the modulation process employed by the transmitter 210. The processed packets are then provided to a decoder 216. The decoder 216 attempts to decode the N packets, and determines, for each non-decoded packet, how many punctured parity bytes are required so that each non-decoded packet is decoded correctly. Each destination station, e.g., the j-th destination station, reports to the origination station R2 different numbers \( S_{1,i}, S_{2,2}, S_{3,3}, \ldots, S_{j,R2} \), where \( S_{ij} \) denotes the number of packets that require only the first i punctured parity bytes in order to be decoded correctly by the destination station. Thus, the total number of erased packets for j-th destination station is:

\[
S_j = \sum_{i=1}^{R2} S_{ij} \tag{14}
\]

The origination station then selects Pj parity bytes of the i-th RS code for each i=1,2,3, ..., R2, where:

\[
Q_j = \max_i \left( \sum_{i=1}^{R2} S_{ij} \right) \tag{15}
\]

The selected Qj parity bytes are provided to the encoder 208. The encoder 208 encodes the selected Qj parity bytes with the code C4. The code C3 is selected so that a packet sent as an element of the code C3 is decoded correctly with a probability \( P_{C3} \) by every destination station. A relationship between a required probability of receiving the message \( P_{\text{Message}} \), the probability \( P_{C2} \), and the probability \( P_{C3} \) is given by the following equation:

\[
P_{\text{Message}} = P_{C2} + P_{C3} \tag{16}
\]

Therefore, given the requirement of message delivery expressed in terms of \( P_{\text{Message}} \), Equation (16) is used for selection of codes C2 and C3.

The purpose of encoding the Qj parity bytes by the code C4 is to deliver the encoded Qj parity bytes with reliability expressed in terms of \( P_{C4} \). Consequently, there is no restriction on the organization of the encoded Qj parity bytes. Thus, the encoded Qj parity bytes may be organized in an arbitrary number of packets. Consequently, in one embodiment, all of the Qj parity bytes form one packet. In another embodiment, the Pj parity bytes are organized into a plurality of packets.

FIG. 4 illustrates a method used by the j-th destination station to recover the erased packets.

In step 402 the packet or packets containing the selected Qj parity bytes of the RS code received by the j-th destination station from the origination station are decoded. Because the packet or packets were protected by the code C4, decoding is always successful. The method continues in step 404.

In step 404, the variable i is initialized to the value 1, and compared against R2. If the value of the variable i is
smaller than R_2', the method continues in step 406; otherwise the method continues in step 412.

[0088] In step 406, the j-th origination station decoder recovers the punctured byte at the first punctured byte position of each erased packet from the first P_1 parity byte of the first RS code. This recovery is always possible because S_1 \leq P_1. The method continues in step 408.

[0089] In step 408, the decoder decodes S_1 packets. The method continues in step 410.

[0090] In step 410, all the missing punctured bytes for the packets decoded in step 408 are computed. The total number of missing punctured bytes at the second punctured byte position is given by S_2 \leq P_2. The method returns to step 404.

[0091] In step 412, the method stops because all packets comprising the message have been computed.

[0092] The total number of extra bytes transmitted by this method relative to the baseline system is:

\[
1 + \frac{R_1 + R_2}{K} \sum_{j} s_j = \left(1 + \frac{R_1 + R_1}{K}\right) \sum_{j} \max\left\{s_j'\right\} 
\]

[0093] Because this number is smaller than the number of bytes (R_1S') of the previous embodiment, a system in accordance with this embodiment may be more efficient.

[0094] Those of skill in the art would understand that information and signals may be represented using any of a variety of different technologies and techniques. For example, data, instructions, commands, information, signals, bits, symbols, and chips that may be referenced throughout the above description may be represented by voltages, currents, electromagnetic waves, magnetic fields or particles, optical fields or particles, or any combination thereof.

[0095] Those of skill would further appreciate that the various illustrative logical blocks, modules, circuits, and algorithm steps described in connection with the embodiments disclosed herein may be implemented as electronic hardware, computer software, or combinations of both. To clearly illustrate this interchangeability of hardware and software, various illustrative components, blocks, modules, circuits, and steps have been described above generally in terms of their functionality. Whether such functionality is implemented as hardware or software depends upon the particular application and design constraints imposed on the overall system. Skilled artisans may implement the described functionality in varying ways for each particular application, but such implementation decisions should not be interpreted as causing a departure from the scope of the present invention.

[0096] The various illustrative logical blocks, modules, and circuits described in connection with the embodiments disclosed herein may be implemented or performed with a General Purpose Processor (GPP), a Digital Signal Processor (DSP), an Application Specific Integrated Circuit (ASIC), a Field Programmable Gate Array (FPGA) or other programmable logic device, discrete gate or transistor logic, discrete hardware components, or any combination thereof designed to perform the functions described herein. A general purpose processor may be a microprocessor, but in the alternative, the processor may be any conventional processor, controller, microcontroller, or state machine. A processor may also be implemented as a combination of computing devices, e.g., a combination of a DSP and a microprocessor, a plurality of microprocessors, one or more microprocessors in conjunction with a DSP core, or any other such configuration.

[0097] The steps of a method or algorithm described in connection with the embodiments disclosed herein may be embodied directly in hardware, in a software module executed by a processor, or in a combination of the two. A software module may reside in RAM memory, flash memory, ROM memory, EEPROM memory, registers, hard disk, a removable disk, a CD-ROM, or any other form of storage medium known in the art. An exemplary storage medium is coupled to the processor such that the processor can read information from, and write information to, the storage medium. In the alternative, the storage medium may be integral to the processor. The processor and the storage medium may reside in an ASIC. The ASIC may reside in a user terminal (presumably previously defined broadly). In the alternative, the processor and the storage medium may reside as discrete components in a user terminal.

[0098] The previous description of the disclosed embodiments is provided to enable any person skilled in the art to make or use the present invention. Various modifications to these embodiments will be readily apparent to those skilled in the art, and the generic principles defined herein may be applied to other embodiments without departing from the spirit or scope of the invention. Thus, the present invention is not intended to be limited to the embodiments shown herein, but is to be accorded the widest scope consistent with the principles and novel features disclosed herein.

[0099] A portion of the disclosure of this patent document contains material which is subject to copyright protection. The copyright owner has no objection to the facsimile reproduction by anyone of the patent document or the patent disclosure, as it appears in the Patent and Trademark Office patent file or records, but otherwise reserves all copyright rights whatsoever.

What is claimed is:

1. A method for multicast transmission of a message, comprising:
   processing each of a plurality of data sets to generate a processed data set and a parity block for each data set;
   processing the parity blocks to generate at least one packet; and
   transmitting the processed data sets and the at least one packet.
2. The method of claim 1 wherein the processing each of the plurality of data sets comprises:
encoding each of the plurality of data sets with a first code to provide a coded data set; and
encoding each of the coded data sets with a systematic code to provide the parity block.
3. The method of claim 1 wherein the processing each of the plurality of data sets comprises:
encoding each of the plurality of data sets with a first code to provide coded data sets; and
puncturing each of the coded data sets to provide the parity block.
4. The method of claim 1 wherein the processing each of the plurality of data sets comprises:
encoding each of the plurality of data sets with a first code; and
encoding each of the plurality of data sets with a second code.
5. The method of claim 4 wherein the encoding each of the plurality of data sets with a first code and the encoding each of the plurality of data sets with a second code comprises:
encoding each of the plurality of data sets with a first code; and
encoding each of the plurality of data sets with a second code;
wherein at least one code is a systematic code.
6. The method of claim 1 wherein the processing the parity blocks comprises:
encoding the parity blocks with a third code having a pre-determined code distance to generate at least one second parity block; and
encoding the at least one second parity block with a fourth code to generate the at least one packet.
7. The method of claim 6 wherein the encoding the parity blocks with a third code having a pre-determined code distance to generate at least one second parity block comprises:
encoding the parity blocks with a systematic code.
8. The method of claim 7 wherein the encoding the parity blocks with a systematic code comprises:
encoding the parity blocks with a Reed-Solomon code.
9. A method for decoding a message, comprising:
receiving a plurality of packets comprising the message, and at least one other packet;
decoding each packet of the plurality of packets comprising the message; and
decoding each incorrectly decoded packet in accordance with the at least one other packet when a number of the incorrectly decoded packets is less than or equal to a pre-determined code distance.
10. The method of claim 9 wherein the receiving at least one other packet comprises:
receiving the at least one other packet comprising a first set of parity bits encoded by a code, the code having a pre-determined code distance.
11. The method of claim 9 wherein the decoding each incorrectly decoded packet, comprises:
calculating for each correctly decoded packet a parity block;
decoding the at least one other packet to generate a first set of parity bits;
calculating for each incorrectly decoded packet an associated parity block from the parity blocks of the correctly decoded packets and the first set of parity bits; and
decoding each of the incorrectly decoded packets from the associated parity blocks.
12. A method for multicast transmission of a message, comprising:
processing each of a plurality of data sets to generate a processed data set and a parity block for each data set;
transmitting the processed data sets as packets;
receiving signals containing information about incorrectly decoded packets, and when the signals are received:
processing the parity blocks to generate at least one packet; and
transmitting the at least one packet.
13. The method of claim 12 wherein the processing each of the plurality of data sets comprises:
encoding each of the plurality of data sets with a first code to provide a coded data set; and
encoding each of the coded data sets with a systematic code to provide the parity block.
14. The method of claim 12 wherein the processing each of the plurality of data sets comprises:
encoding each of the plurality of data sets with a systematic code to provide the parity block.
15. The method of claim 12 wherein the processing each of the plurality of data sets comprises:
encoding each of the plurality of data sets with a first code to provide coded data sets; and
puncturing each of the coded data sets to provide the parity block.
16. The method of claim 15 wherein the encoding each of the plurality of data sets with a first code and the encoding each of the plurality of data sets with a second code comprise:
encoding each of the plurality of data sets with a first code; and
encoding each of the plurality of data sets with a second code;
wherein at least one code is a systematic code.
17. The method of claim 12 wherein the processing the parity blocks to generate at least one packet comprises:
selecting a set of bits from the parity blocks in accordance with the received signals; and
encoding the selected set to generate the at least one packet.

18. The method of claim 17 wherein the selecting a set of bits comprises:

determining a first number of the incorrectly decoded packets from each of the received signals;
determining a largest of the determined first numbers; and
selecting a second number of parity bits for the set from the parity blocks, the second number being equal to a product of the largest determined number and a number of bit positions in the parity block.

19. The method of claim 17 wherein the encoding the selected set comprises:

encoding the selected set with a third code having a pre-determined code distance to generate at least one second parity block; and

encoding the at least one second parity block with a fourth code to generate the at least one packet.

20. The method of claim 19 wherein the encoding the selected set with a third code having a pre-determined code distance to generate at least one second parity block comprises:

encoding the selected set with a systematic code.

21. The method of claim 20 wherein the encoding the selected set with a systematic code comprises:

encoding the selected set with a Reed-Solomon code.

22. The method of claim 17 wherein the selecting a set of bits comprises:

determining, from each of the received signals, a set of numbers, wherein a size of each of the sets of numbers is equal to a size of a set of identifiers, wherein each identifier identifies one bit position in each of the parity blocks, and wherein each identifier identifies one number in each of the sets of numbers;
determining, for each identifier, a largest number of the numbers identified by the identifier;
providing, for each identifier, a first set of bits comprising bits identified by the identifier in each of the parity blocks;
encoding each of the first sets of bits with a third code having a pre-determined code distance to provide a second set of bits; and
selecting, from parity bits of each of the second sets of bits a second number of bits, wherein the second number of bits is equals to the largest determined number identified by the identifier.

23. The method of claim 22 wherein the encoding each of the first sets of bits with a third code having a pre-determined code distance to provide a second set of bits comprises:

encoding each of the first set with a systematic code.

24. The method of claim 23 wherein the encoding each of the first sets of bits with a systematic code comprises:

encoding each of the first sets of bits with a Reed-Solomon code.

25. The method of claim 17 wherein the encoding the selected set comprises:

encoding the selected set with a fourth code to generate the at least one packet.

26. A method for decoding a message, comprising:

receiving a plurality of packets comprising the message;
decoding each packet of the received plurality of packets comprising the message;
transmitting a signal containing at least one number;
receiving at least one packet in response to the signal; and
decoding the incorrectly decoded packets in accordance with the received at least one packet.

27. The method of claim 26 wherein the transmitting a signal containing at least one number comprises:

transmitting a signal containing a number of incorrectly decoded packets.

28. The method of claim 26 wherein the transmitting a signal containing at least one number comprises:

transmitting a signal containing a set of numbers.

29. The method of claim 26 wherein the transmitting a signal containing at least one number comprises:

transmitting a signal containing at least one number when a number of incorrectly decoded packet is less than or equal to a code distance.

30. The method of claim 26 wherein the receiving at least one other packet comprises:

receiving the at least one other packet comprising a first set of parity bits encoded by a code, the code having a pre-determined code distance.

31. The method of claim 26 wherein the decoding each incorrectly decoded packet, comprises:

calculating for each correctly decoded packet a parity block;

decoding the at least one other packet to generate a first set of parity bits;
calculating for each incorrectly decoded packet an associated parity block from the parity blocks of the correctly decoded packets and the first set of parity bits; and

decoding each of the incorrectly decoded packets from the associated parity blocks.

32. The method of claim 26 wherein the decoding each incorrectly decoded packet, comprises:

decoding the at least one other packet to generate a first set of parity bits;
decoding a subset of incorrectly decoded packet in accordance with the first set of parity bits;
calculating for each packet from the decoded subset an associated parity block; and

repeating the decoding a subset of incorrectly decoded packet in accordance with the first set of parity bits and the calculating for each packet from the decoded subset an associated parity block until all incorrectly decoded packets are decoded.
33. An apparatus for multicast transmission of a message, comprising:

a processor;

a storage medium coupled to the processor and containing a set of instructions executable by the processor to:

process each of a plurality of data sets to generate a processed data set and a parity block for each data set;

process the parity blocks to generate at least one packet; and

a transmitter coupled to the processor and configured to transmit the processed data sets and the at least one packet.

34. The apparatus of claim 33 wherein the processor processes each of a plurality of data sets to generate a processed data set and a parity block for each data set by executing a set of instructions to:

encode each of the plurality of data sets with a first code to provide a coded data set; and

encode each of the coded data sets with a systematic code to provide the parity block.

35. The apparatus of claim 33 wherein the processor processes each of a plurality of data sets to generate a processed data set and a parity block for each data set by executing a set of instructions to:

encode each of the plurality of data sets with a first code to provide coded data sets; and

puncture each of the coded data sets to provide the parity block.

36. The apparatus of claim 33 wherein the processor processes each of a plurality of data sets to generate a processed data set and a parity block for each data set by executing a set of instructions to:

encode each of the plurality of data sets with a first code; and

encode each of the plurality of data sets with a second code;

37. The apparatus of claim 36 wherein the processor encode each of the plurality of data sets with a first code and encode each of the plurality of data sets with a second code by executing a set of instructions to:

encode each of the plurality of data sets with a first code; and

encode each of the plurality of data sets with a second code;

wherein at least one code is a systematic code.

38. The apparatus of claim 33 wherein the processor processes the parity blocks by executing a set of instructions to:

encode the parity blocks with a third code having a pre-determined code distance to generate at least one second parity block; and

encode the at least one second parity block with a fourth code to generate the at least one packet.

39. The apparatus of claim 38 wherein the processor encodes the parity blocks with a third code having a pre-determined code distance to generate at least one second parity block by executing a set of instructions to:

encode the parity blocks with a systematic code.

40. The apparatus of claim 39 wherein the processor encodes the parity blocks with a systematic code by executing a set of instructions to:

encode the parity blocks with a Reed-Solomon code.

41. An apparatus for decoding a message, comprising:

a receiver configured to receive a plurality of packets comprising the message and at least one other packet; a processor, and

a storage medium coupled to the processor and containing a set of instructions executable by the processor to:

decode each packet of the plurality of packets comprising the message; and

decode each incorrectly decoded packet in accordance with the at least one other packet when a number of the incorrectly decoded packets is less than or equal to a pre-determined code distance.

42. The apparatus of claim 41 wherein the at least one other packet comprises:

a first set of parity bits encoded by a code, the code having a pre-determined code distance.

43. The apparatus of claim 41 wherein the processor decodes each incorrectly decoded packet by executing a set of instructions to:

calculate for each correctly decoded packet a parity block;

decode the at least one other packet to generate a first set of parity bits;

calculate for each incorrectly decoded packet an associated parity block from parity blocks of the correctly decoded packets and the first set of parity bits; and

decode each of the incorrectly decoded packets from the associated parity blocks.

44. A apparatus for multicast transmission of a message, comprising:

a processor;

a storage medium coupled to the processor and containing a set of instructions executable by the processor to:

process each of a plurality of data sets to generate a processed data set and a parity block for each data set; and

process the parity blocks to generate at least one packet;

a transmitter, communicatively coupled to the processor, configured to:

transmit the processed data sets as packets; and

transmit the at least one packet; and

a receiver, communicatively coupled to the processor, configured to:

receive signals containing information about incorrectly decoded packets; and

provide the information to the processor.
45. The apparatus of claim 44 wherein the processor processes each of a plurality of data sets to generate a processed data set and a parity block for each data set by executing a set of instructions to:

    encode each of the plurality of data sets with a first code to provide a coded data set; and

    encode each of the coded data sets with a systematic code to provide the parity block.

46. The apparatus of claim 44 wherein the processor processes each of a plurality of data sets to generate a processed data set and a parity block for each data set by executing a set of instructions to:

    encode each of the plurality of data sets with a first code to provide coded data sets; and

    puncture each of the coded data sets to provide the parity block.

47. The apparatus of claim 44 wherein the processor processes each of a plurality of data sets to generate a processed data set and a parity block for each data set by executing a set of instructions to:

    encode each of the plurality of data sets with a first code; and

    encode each of the plurality of data sets with a second code.

48. The apparatus of claim 47 wherein the processor encodes each of the plurality of data sets with a first code and encode each of the plurality of data sets with a second code by executing a set of instructions to:

    encode each of the plurality of data sets with a first code; and

    encode each of the plurality of data sets with a second code;

wherein at least one code is a systematic code.

49. The apparatus of claim 44 wherein the processor processes the parity blocks to generate at least one packet by executing a set of instructions to:

    select a set of bits from the parity blocks in accordance with the information provided by the receiver; and

    encode the selected set to generate the at least one packet.

50. The apparatus of claim 49 wherein the processor selects a set of bits by executing a set of instructions to:

    determine a first number of the incorrectly decoded packets from each of the received signals;

    determine a largest of the determined first numbers; and

    select a second number of parity bits for the set, the second number being equal to a product of the largest determined number and a number of bit positions in the parity block.

51. The apparatus of claim 49 wherein the processor encodes the selected set by executing a set of instructions to:

    encode the selected set with a third code having a pre-determined code distance to generate at least one second parity block; and

    encode the at least one second parity block with a fourth code to generate the at least one packet.

52. The apparatus of claim 51 wherein the processor encodes the selected set with a third code having a pre-determined code distance to generate at least one second parity block by executing a set of instructions to:

    encode the selected set with a systematic code.

53. The apparatus of claim 52 wherein the processor encodes the selected set with a systematic code by executing a set of instructions to:

    encoding the selected set with a Reed-Solomon code.

54. The apparatus of claim 49 wherein the processor selects a set of bits by executing a set of instructions to:

    determine, from each of the received signals, a set of numbers, wherein a size of each of the sets of numbers is equal to a size of a set of identifiers, wherein each identifier identifies one bit position in each of the parity blocks, and wherein each identifier identifies one number in each of the sets of numbers;

    determine, for each identifier, a largest number of the numbers identified by the identifier;

    provide, for each identifier, a first set of bits comprising bits identified by the identifier in each of the parity blocks;

    encode each of the first sets of bits with a third code having a pre-determined code distance to provide a second set of bits; and

    select, from parity bits of each of the second sets of bits a second number of bits, wherein the second number of bits is equals to the largest determined number identified by the identifier.

55. The apparatus of claim 54 wherein the processor encodes each of the first sets of bits with a third code having a pre-determined code distance to provide a second set of bits by executing a set of instructions to:

    encode each of the first sets of bits with a systematic code.

56. The apparatus of claim 55 wherein the processor encodes each of the first sets of bits with a systematic code by executing a set of instructions to:

    encode each of the first sets of bits with a Reed-Solomon code.

57. The apparatus of claim 49 wherein the processor encodes the selected set by executing a set of instructions to:

    encode the selected set with a fourth code to generate the at least one packet.

58. A apparatus for decoding a message, comprising:

    a receiver configured to:

    receive a plurality of packets comprising the message; and

    receive at least one packet;

    a processor, coupled to the receiver;

    a storage medium coupled to the processor and containing a set of instructions executable by the processor to:

    decode each packet of the received plurality of packets comprising the message; and

    decode each incorrectly decoded packet in accordance with the received at least one packet; and
a transmitter, coupled to the processor, configured to transmit a signal containing number of incorrectly decoded packets;

59. The apparatus of claim 58 wherein the transmitter is further configured to transmit the signal containing a number of incorrectly decoded packets when the number of the incorrectly decoded packet is less than or equal to a code distance.

60. The apparatus of claim 58 wherein the receiver is configured to receive the at least one other packet comprising a first set of parity bits encoded by a code, the code having a pre-determined code distance.

61. The apparatus of claim 58 wherein the processor decodes each incorrectly decoded packet, by executing a set of instructions to:
   calculate for each correctly decoded packet a parity block;
   decode the at least one other packet to generate the first set of parity bits;
   calculate for each incorrectly decoded packet an associated parity block from the parity blocks of the correctly decoded packets and the first set of parity bits; and
   decode each of the incorrectly decoded packets from the associated parity blocks.

62. The apparatus of claim 58 wherein the processor decodes each incorrectly decoded packet, by executing a set of instructions to:
   decode the at least one other packet to generate a first set of parity bits;
   decode a subset of incorrectly decoded packet in accordance with the first set of parity bits;
   calculate for each packet from the decoded subset an associated parity block; and
   repeat the decoding a subset of incorrectly decoded packet in accordance with the first set of parity bits and the calculating for each packet from the decoded subset an associated parity block until all incorrectly decoded packets are decoded.

* * * * *