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# WRITING AND READING ON PATTERNED MEDIA

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## Introduction

Patterned media offers the potential of ultra high density recording but is resplendent with new and challenging technical problems that give smiles to the researcher and tears to the engineer. Not the least of these problems is the reliable writing and the reading on this media.

A rapidly growing body of literature exists on various problems related to signal processing for patterned media [1-15]. In this report we give a very brief introduction to some of the problems that our group at CMRR has been pursuing on this subject.

## Problems Associated With Reliable Writing of Digital Data

In current HDD's with disks covered with a continuous magnetic coating, a bit can be written anywhere on the disk. Of course, there is some sanity in the placement of these bits. This sanity is enforced by servo markers and sector headers which define the positions of tracks and sectors. In writing, once the position of a sector and a track are found, the placement of the bits on this track is governed by the write clock. This same clock is used in reading. In reading, variations in the angular velocity of the spinning disk are compensated for by timing recovery and tracking circuitry.

For patterned media, bits can only be written on magnetic islands which are embossed into the disk. Thus the placement of the written bits is governed not by a clock but by the locations of the magnetic islands on the disk. This phenomenon introduces a number of new difficulties not present in continuous media. For example, even if the islands were laid out in a perfect geometric pattern, in order to write a bit on a given island, the write head would have to identify the exact position of the island in question. This problem is exacerbated by the fact that there will be irregularities in the placement of the islands. Thus, even if one knew the exact placement of a given island, one may not know the position of other neighboring islands.

As a result, errors may occur in the write process. These errors can take the form of inverted bits (i.e., errors), extra bits (i.e., insertions) or missing bits (i.e., deletions). Relatively little is known about the design and performance of efficient error correcting codes that can correct errors, insertions and deletions. One avenue of research we are pursuing is the search for good codes that correct such imperfections.

## Problems Associated With Reliable Reading of Digital Data

Even if ways are found for reliably writing digital data on patterned media, many problems exist for reliably reading this data.

Consider, for example, the effect of an island being shifted from its nominal position by a shift of $\delta x$ and $\delta z$ in the down-track and cross-track direction respectively. (See Figure 1.)

Assume that these shifts are statistically independent Gaussian random variables of zero mean and variance $\sigma^2$. Let the read-back voltage induced in a read-head centered at (x,z) by an island centered at $(x+\delta x, z+\delta z)$ be expressed in the Taylor series expansion:

$$V(x+\delta x, z+\delta z) = V(x,z) + \delta x V_x(x,z) + \delta z V_z(x,z) + \tfrac{1}{2}[(\delta x)^2 V_{xx}(x,z) +$$

$$2\delta x \delta z V_{xz}(x,z) + (\delta z)^2 V_{zz}(x,z)] + \zeta(x,z)$$

Here, the subscripts indicate partial derivatives, and $\zeta(x,z)$ is the modeling error due to representing the read-back voltage with the first and second derivative. Define e(x,z) as:

$$e(x,z) = \delta x V_x(x,z) + \delta z V_z(x,z) + \tfrac{1}{2}[(\delta x)^2 V_{xx}(x,z) + 2\delta x \delta z V_{xz}(x,z) + (\delta z)^2 V_{zz}(x,z)].$$

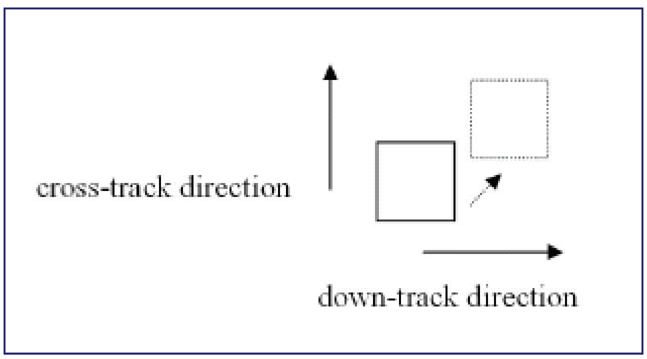
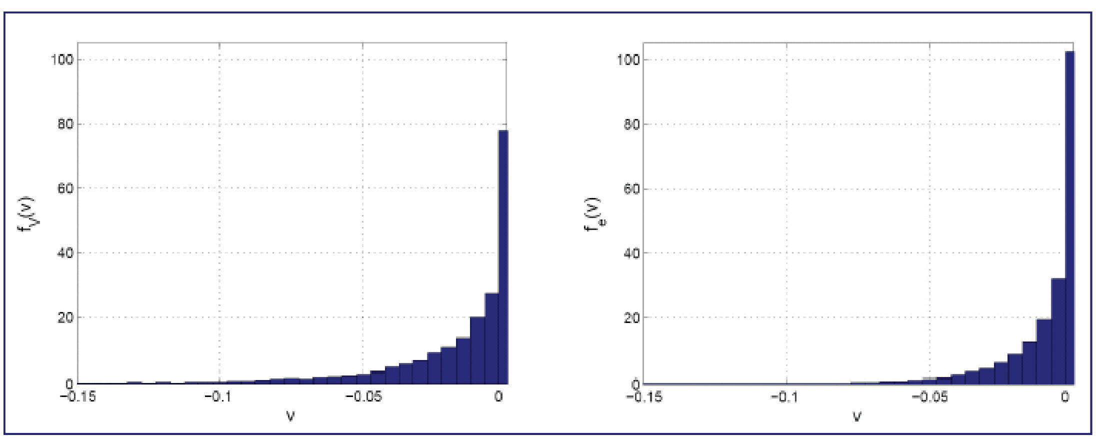



Figure 1.




6000 samples, 6 hours

76582 samples, 1 minute

Figure 2.

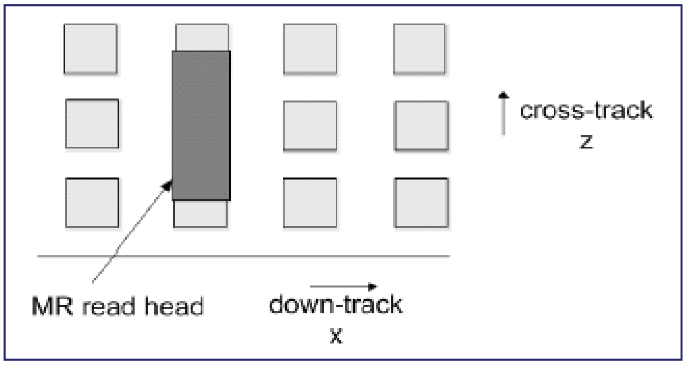



Figure 3.

Then e(x,z) is the second order approximation to the island jitter noise, while the true jitter noise is given as:

$$[V(x+\delta x, z+\delta z) - V(x,z)].$$

One might wonder why a second order approximation was used. The answer is that a second order approximation gives a very good approximation to the true jitter noise and is much easier to compute than the true value. A comparison of the probability density function of the second order approximation and probability density function of the true jitter noise is shown in Figure 2 for a particular choice of the function V(x,z). On the other hand, a first order approximation (using only the terms $\delta x V_x(x,z)$ and $\delta z V_z(x,z)$) would not suffice since the derivatives $V_x(x,z)$ and $V_z(x,z)$ are zero. Furthermore, even if these derivatives were not zero, the terms $\delta x$ and $\delta z$ would predict that the jitter noise is Gaussian which most certainly it is not!

Another problem which we have considered concerns the situation where the read-head cannot be made as small as a single island so that islands on more than a single track induce voltages in the read-head. In Figure 3, we show the case where the read-head is centered over a middle track but where the islands on a total of 3 tracks induce voltages on the read-head. In what follows we will assume this special case.

Again we assume there is no error in writing any desired pattern on the islands. The usual approach in reading is to attempt to read the data on the middle track while considering the voltages induced by the data on the two neighboring tracks as noise. Such a situation is called detection with inter-track interference. If the inter-track interference is comparable to the desired signal from the middle track, even the best detector will have poor performance.

Here we consider a case where the read signal induced by the two interfering tracks is of the same order of magnitude as the signal induced by the middle track. Instead of thinking of the signal from the neighboring interfering tracks as noise, the entire output signal from the read-head (due to all three tracks) is considered as information. The good news is that we are reading the data from all three tracks simultaneously so that we have a big boost to the throughput from the read-head. The bad news is that certain patterns of recorded channel bits on the three tracks give identical outputs from the read-head so that only a subset of these patterns can be used, thus reducing the information storing capacity of the disk.

It is best to understand our approach by a simple example. In this simple example we will ignore intersymbol interference (ISI) along the tracks but the reader should be assured that this assumption is not a limitation to the technique but rather is made to simplify the explanation. Let us denote the three channel bits recorded in position i on the three tracks as $U_i$, $M_i$, $L_i$, where $U_i$, $M_i$, $L_i$ each take on the values -1 and +1. The letters U, M and L were chosen to stand for Upper, Middle, and Lower (track). We assume that the read-head linearly combines the signal from the three tracks so that the noiseless output of this head, $Y_i$, corresponding to these recorded bits is given as:

$$Y_i = G_U U_i + G_M M_i + G_L L_i.$$

The quantities $G_U$, $G_M$, and $G_L$ are constants which can be computed from a micromagnetic model of the head.

In what follows we normalize the noiseless output by setting $G_M = 1$. Further, we assume that the head is symmetric so that $G_U = G_L = \gamma$. Thus the noiseless output is given as:

$$Y_i = \gamma(Ui + L_i) + M_i.$$

Table I shows the 8 possible recorded channel bit patterns for ($U_i$, $M_i$, $L_i$) and the corresponding values for the noiseless output $Y_i$.

Note that if $\gamma = 0$, there are only two distinct noiseless outputs, -1 and +1, corresponding to whether the middle recorded channel bit is a -1 or a +1. This is the case of no inter-track interference where the read-head sees only one track.

If $\gamma$ is less than 1/2, the noiseless output will be negative whenever the middle bit is negative and will be positive whenever the middle bit is positive. Thus, in the absence of noise one could detect the middle bit with perfect reliability by observing the sign of the output, but in the presence of noise the reliability of this decision rapidly decreases as $\gamma$ increases.

Table I : Recorded Channel Bits and Noiseless Outputs

| $U_i$ | $M_i$ | $L_i$ | $Y_i$ |
|---|---|---|---|
| -1 | -1 | -1 | $-1 - 2\gamma$ |
| -1 | -1 | +1 | -1 |
| +1 | -1 | -1 | -1 |
| +1 | -1 | +1 | $-1 + 2\gamma$ |
| -1 | +1 | -1 | $+1 - 2\gamma$ |
| -1 | +1 | +1 | +1 |
| +1 | +1 | -1 | +1 |
| +1 | +1 | +1 | $+1 + 2\gamma$ |

If $\gamma$ is greater than 1/2, note that the polarity of the noiseless output is not always the same as the polarity of the channel bit on the middle track. For example, if a +1 is recorded on the middle track and if two -1's are recorded on the upper track and the lower track, the noiseless output from the read-head would be:

$$Y_i = +1 - 2\gamma$$

which is negative for $\gamma$ greater than 1/2. Thus a simple threshold detector which assumes that the polarity of $Y_i$ is equal to the polarity of the bit to be detected sometimes would fail in detecting the polarity middle bit <u>even in the absence of any noise</u>.

The number of distinct noiseless outputs depends on the value of $\gamma$. This dependence is summarized in Table II.

Table II: Number of Distinct Noiseless Outputs for Different Values of $\gamma$.

| Value of $\gamma$ | Number of Distinct Noiseless Outputs |
|---|---|
| 0 | 2 |
| $0 < \gamma < 1/2$ | 6 |
| 1/2 | 5 |
| $1/2 < \gamma < 1$ | 6 |
| 1 | 4 |

To understand the consequences of this table, let us consider in more detail the case where $\gamma = 1$. This case corresponds to a read-head that spans all 3 tracks and has essentially the same response for all three tracks. The noiseless response to all possible values of ($U_i$, $M_i$, $L_i$) is given in Table III.

Table III: Noiseless Outputs for $\gamma = 1$

| | | | Noiseless Read-Head |
|---|---|---|---|
| -1 | -1 | -1 | -3 |
| -1 | +1 | -1 | -1 |
| +1 | -1 | -1 | -1 |
| -1 | -1 | +1 | -1 |
| +1 | -1 | +1 | +1 |
| -1 | +1 | +1 | +1 |
| +1 | +1 | -1 | +1 |
| +1 | +1 | +1 | +3 |

Note that all three patterns containing a single +1 have the same noiseless output so that a detector could not differentiate between them even in the absence of any noise. The same is true for the three patterns containing two +1's. This suggests using only one of the three patterns containing one +1, say (-1, +1, -1), and one of the three patterns containing two +1s, say (+1, -1, +1), in recording. We could then write using one of the four patterns,

$$\{(-1, -1, -1), \quad (-1, +1, -1), \quad (+1, -1, +1), \quad (+1, +1, +1)\}$$

on the three tracks to store two bits of information. Thus in the absence of noise, a detector observing the output from the read-head could correctly detect these two information bits. If noise is present, errors will occur but the probability of error will depend upon the signal-to-noise ratio. If the noise is additive and Gaussian with mean 0 and variance $\sigma^2$, it is easy to compute the probability of error in detecting a symbol consisting of the two information bits:

$$\Pr[\text{symbol error}] = 3/2Q(1/\sigma^2).$$

Here, $Q(\alpha)$ is the area under a normalized (zero mean, unit variance) Gaussian from $\alpha$ to infinity. The probability of bit error is bounded as:

$$1/2 \Pr[\text{symbol error}] < P[\text{bit error}] < \Pr[\text{symbol error}]$$

where if one uses a Gray code to map the two information bits to the four channel patterns, the lower bound is tight at high signal-to-noise ratio.

We next consider the case of a read-head that does not extend fully over the upper and lower island. This is the situation shown in Figure 1. In such a case, $\gamma < 1$. As shown in Table I, for all value of $\gamma$ in the range $0 < \gamma < 1$, except for $\gamma = 1/2$, there are six distinct noiseless outputs. One can easily choose six of the input patterns to yield these six distinct noiseless outputs. Mapping information to these six patterns, one can see that the information storage capacity in this case is $(1/3)\log_2(6) = 0.8617$ bits per island. The case of $\gamma = 1/2$ is special in that the two noiseless outputs $(-1+2\gamma)$, and $(+1-2\gamma)$ coincide in which case there are only five distinct noiseless outputs and the capacity is $(1/3)\log_2(5) = 0.7740$ bits per island. The performance in additive white Gaussian noise depends heavily on the value of $\gamma$. An interesting choice for $\gamma$ is $\gamma = 1/3$ in which case the six noiseless output values are: {-5/3, -3/3, -1/3, +1/3, +3/3, +5/3} which are uniformly spaced on the real line. The probability of symbol error in this case is:

$$\Pr[\text{symbol error}] = 5/3Q(1/3\sigma^2).$$

It is easy to obtain an exact expression for the probability of symbol error for an arbitrary value of $\gamma$ but the result is not as pretty. Care should be taken in comparing the formulas obtained for the probability of symbol errors for different values of $\gamma$ since the peak value of the noiseless output signal, the variance of the Gaussian noise and the symbol size may all, like $\gamma$ itself, be a function of the read-head characteristics.

The previous description summarizes some of the research performed by Hao Wang, Seyhan Karakulak, H. Neal Bertram, Paul H. Siegel and myself on signal processing for patterned media. Interested readers should attend our CMRR review where more up-to-date information is described in much greater detail.

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