Nonlinear dynamic modeling of a thermal actuator in a hard disk drive via Hammerstein system identification

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Thermal actuator in a hard disk drive for flying height control
A Hammerstein system has a block oriented structure where static input nonlinearity $f(\cdot)$ and a linear dynamic system $G(q)$ are separated.
Motivation

- Understanding the nonlinear behavior of the thermal protrusion actuator → effective flying height control.
- Quadratic relationship between power and voltage for a resistance component + nonlinearity due to height dependent thermal conductivity.
- Hammerstein structure → efficient in modeling systems with actuator nonlinearity.
Input/output data from a thermal actuator

Figure: Concatenated input signal $u(t)$ (left) and concatenated measured output $z(t)$ (right).
A piecewise linear approximation of $f(\cdot)$ using triangle basis functions.

- Center location vector $m = [m_1 \cdots m_M]^T$.
- Amplitude vector $\mu = [\mu_1 \cdots \mu_M]^T$.

The static nonlinearity $f(\cdot)$ can be arbitrarily well approximated with a dense grid of triangle basis functions.

$$
\sup_{u(t) \in [u_{\min}, u_{\max}]} \lim_{M \to \infty} \sum_{m=1}^{M} |\mu_m f_m(u(t)) - f_0(u(t))| = 0.
$$
Let $g(i), \ i = 0, 1, \cdots$ be the causal sequence of unit impulse responses for $G(q)$. The relationship between $x(t)$ and $z(t)$ can be described by

$$z(t) = \sum_{i=0}^{\infty} g(i)x(t - i) + d(t).$$

Let

$$X = \begin{bmatrix}
\hat{x}(1) & \hat{x}(0) & \cdots & \hat{x}(2-N) \\
\hat{x}(2) & \hat{x}(1) & \cdots & \hat{x}(1-N) \\
\vdots & \vdots & \ddots & \vdots \\
\hat{x}(N) & \hat{x}(N-1) & \cdots & \hat{x}(1)
\end{bmatrix}$$

and

$$g = [g(0) \ g(1) \ \cdots \ g(N-1)]^T.$$

Then the estimate of the output $\hat{z}$ can be written as

$$\hat{z} = \begin{bmatrix}
\hat{z}(1) & \hat{z}(2) & \cdots & \hat{z}(N)
\end{bmatrix}^T = Xg.$$
Let $\theta = [g^T \mu^T]^T$. With $\theta$, let us define a positive semidefinite symmetric matrix $\Theta = \theta \theta^T$ as

$$
\Theta = \begin{bmatrix}
    g(0)^2 & \cdots & g(0)\mu(M) \\
    \vdots & \ddots & \vdots \\
    g(N-1)g(0) & \cdots & g(N-1)\mu(M) \\
    \mu(1)g(0) & \cdots & \cdots \mu(1)\mu(M) \\
    \vdots & \ddots & \vdots \\
    \mu(M)g(0) & \cdots & \mu(M)^2
\end{bmatrix}.
$$

For the purpose of normalization, the following condition will be applied to the system parameters of linear dynamics:

$$
\sum_{k=1}^{N} \Theta(k) = g(0)^2 + \cdots + g(N)^2 = \alpha
$$
A non-negative staircase voltage input → a non-negative monotonically increasing condition on the system parameters of the input static nonlinearity

$$0 \leq \mu_1 < \cdots < \mu_M.$$ 

Due to the structure of $\Theta$, the monotonically increasing condition of $\mu$ is relaxed to

$$0 \leq \Theta(N+1, N+1) < \cdots < \Theta(N+M, N+M)$$

which is equivalent to

$$0 \leq \mu_1^2 < \cdots < \mu_M^2.$$
Model output $\hat{z}$ is defined by

$$\hat{z} = \min(t, N) \sum_{k=1}^{\min(t, N)} T_{t-k+1, k}$$

where

$$T = \rho \mu g^T$$

$$= \rho \Theta(N + 1 : N + M, 1 : N).$$

and

$$\rho = [\rho(u(1)) \cdots \rho(u(N))]^T$$

where

$$\rho(u(t)) = \begin{bmatrix} \cdots & 0 & \frac{m_{k+1} - u(t)}{m_{k+1} - m_k} & \frac{u(t) - m_k}{m_{k+1} - m_k} & 0 & \cdots \end{bmatrix}$$
Consider

variable $\Theta$ to define

$$\hat{z} = \min_{t,N} \sum_{k=1}^{\min(t,N)} T_{t-k+1,k}$$

where $T = \rho \Theta(N + 1 : N + M, 1 : N)$

Minimize

$$w_1 ||z(t) - \hat{z}(t)||_2 + w_2 \text{trace}(\Theta)$$

subject to

$$\sum_{k=1}^{N} \Theta(k) = \alpha \text{ and}$$

$$0 \leq \Theta(N + 1, N + 1) < \cdots < \Theta(N + M, N + M).$$
Parameter separation

- Need to separate the parameters of the linear dynamical system $g$ and the parameter of the static nonlinearity $\mu$.
- Singular Value Decomposition (SVD) of $\Theta$

$$\Theta = U\Sigma V^T$$

$U$ and $V$ are orthogonal matrices ($U = V$ due to the structure of $\Theta$) and $\Sigma$ is a rectangular diagonal matrix (positive diagonal entries of $\Sigma$ are called singular values).

- From the parameter vector $\theta = [g^T \mu^T]^T$, where $\Theta = \theta \theta^T$

  $$\theta = \sqrt{\sigma_1}U(:, 1)$$
  $$g = \theta(1 : N)$$
  $$\mu = \theta(N + 1 : N + M)$$
Identification of a thermal actuator based on experimental data

Figure: Identified input static nonlinear block (left) and linear dynamic block (right).
Identification of a thermal actuator based on experimental data

Figure: Concatenated input signal $u(t)$ (10 × magnified, black, left) and estimated intermediate signal $\hat{x}(t)$ (red, left). Concatenated measured output $z(t)$ (black, right) and simulated output $\hat{z}(t)$ (red, right).
Figure: Simulated outputs from the Hammerstein system (red) and only from the linear dynamic system (black).
Conclusions

- Nonlinear dynamics of a thermal actuator in a hard disk drive → Hammerstein system.
- When the voltage input is low → the quadratic relationship between the input voltage and the intermediate power signal
- As the input increases (the read/write head gets close to the disk) → nonlinearity due to the head’s extreme proximity to the disk.
- The resulting model can be used efficiently to predict the behavior of the thermal protrusion actuator for flying height compensation.