Control Oriented Modeling of Product Variabilities in Dual-Stage Suspensions

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motivation

- Dual Stage suspensions are used in EHDR
- manufacturing tolerances, operating conditions

Servo controllers must be robust to uncertainty.
Uncertainty Modeling (nominal model + uncertainty description)

- use experimental data
- structured uncertainty models (reduce conservativeness)
- let the data speak (identification must reveal structure of uncertainty)
- Instead of one model have a set of models

\[ M = \{ G | G = F(\theta, \delta) \delta \in [-1,1] \} \]

Issues: - model order
- number of parameters
- size of perturbations

\[ \{ \text{dynamics}, \text{uncertainty} \} \]
contents

- working data set
- uncertainty models
  - parametric nominal model with additive perturbations
  - identification of model and uncertainty parameters
- reduced complexity uncertainty models
  - parameter dependency
  - correlation between parameters
- conclusions
case study

- Piezoelectric based micro-actuator.
- Hutchinson Tech. Inc design
working data set

- data composed of 36 measurements
- system response dominated by three modes
- uncertainty mainly present in three modes

<table>
<thead>
<tr>
<th>mode</th>
<th>frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCM</td>
<td>25Hz</td>
</tr>
<tr>
<td>Eblk sway</td>
<td>5kHz</td>
</tr>
<tr>
<td>Sus. sway</td>
<td>9kHz</td>
</tr>
</tbody>
</table>
modeling procedure

- parameter estimation
  - define a low order model and estimate unknown parameters
  - choice of nominal model
- parameter dependency
  - find structure of uncertainty
  - determine size of perturbation
- reduced parameter model
parametric models

\[ G = \frac{\theta_1 z^{-3} + \theta_2 z^{-4} + \theta_3 z^{-5} + \theta_4 z^{-6}}{(1 + \theta_5 z^{-1} + \theta_6 z^{-2})(1 + \theta_7 z^{-1} + \theta_8 z^{-2})(1 + \theta_9 z^{-1} + \theta_{10} z^{-2})} \]

- \( G \) – plant model with 3 modes:
  - keep model order low (6th order model)
  - most of the uncertainty is expected in the natural frequencies and damping

- 10 parameter plant with unknowns
  \[ \theta_k = \theta_k^0 + w_k \delta_k \quad \delta_k \in [-1,1] \]

Objective:
1) find \( \theta_k^0 \) and \( w_k \) for \( k = 1, \ldots, 10 \)
2) reduce the number of independent \( \delta_k \)
**parameter estimation**

- Parameters $\theta_k$ are determined for each of the 36 data sets (least squares fitting)

- Each $\theta_k$ can assume 36 different values (parameter uncertainty) between $[\theta_k^{\text{min}}, \theta_k^{\text{max}}]$

- Question: which choice of nominal parameters $\theta_k$ gives the smallest uncertainty bound $w_k$?

$$\theta_k = \theta_k^0 + w_k \delta_k \quad \delta_k \in [-1,1]$$

- Optimization of $\theta^0$ in the worst case sense:

$$\theta_0^* = \arg \min_{\theta_0} \max_{n=1:36} |\theta^n - \theta_0| \quad w_k = \max_{n=1:36} |\theta^n - \theta_0^*|$$

- Solution:

$$\theta_k^0 = \frac{1}{2}(\theta_k^{\text{max}} + \theta_k^{\text{min}}) \quad w_k = \frac{1}{2}(\theta_k^{\text{max}} - \theta_k^{\text{min}})$$
The choice \( \theta_k = \theta_k^0 + w_k \delta_k \), \( \delta_k \in [-1,1] \) assumes that every parameter is allowed to vary independently.

In mechanical systems it is likely that there exists correlation between poles and zeros.

Making parameter perturbations dependent on each other can reduce uncertainty and reduce the number of parameter perturbations.

**First Step:** normalize parameters so that they can only assume values between \([-1,1]\]

\[
\tilde{\theta}_k = \frac{\theta_k - \theta_k^0}{w_k} \implies \tilde{\theta}_k \in [-1,1]
\]

**Second Step:** find correlation via linear regression.
In order to find dependency between parameters, a linear regression is performed:

\[ e = b_1 - Ax \]
\[ b_k = [\tilde{\theta}_k^1, \tilde{\theta}_k^2, \ldots, \tilde{\theta}_k^{36}]^T \]
\[ A = [b_2, b_3, \ldots, b_{10}, I] \]
\[ x = (A^T A)^{-1} A^T b_1 \]

If the elements of \( x \) are close to 1 or -1, then dependency exists between the parameters.
**parameter dependency results**

- Regression results: two sets of parameters can be grouped together using two "master" parameters $\xi_1$ and $\xi_2$

- $\theta_1, \theta_2, \theta_3, \theta_4$ and $-\theta_7$

- $\theta_9$ and $-\theta_{10}$

\[
\theta_k = \theta_k^0 + w_k \xi_1 \quad k = 1, 2, 3, 4, 7
\]

\[
\theta_k = \theta_k^0 + w_k \xi_2 \quad k = 9, 10
\]
a reduced 5 parameter uncertainty model of 6th order is obtained from parameter dependency analysis

\[
\begin{align*}
\theta_k &= \theta_k^0 + w_k \xi_k \quad k = 1, 2, 3, 4, 7 \\
\theta_k &= \theta_k^0 + w_k \xi_k \quad k = 9, 10 \\
\theta_5 &= \theta_5^0 + w_5 \xi_3 \\
\theta_6 &= \theta_6^0 + w_6 \xi_4 \\
\theta_8 &= \theta_8^0 + w_8 \xi_5 
\end{align*}
\]
modeling results

- In order to validate the model each $\xi_k$ is taken at its extreme points so as to inspect the boundaries.

- exploring the boundaries $\xi = [-1,1]$
interpretation of perturbations

- each parameter perturbation term $\xi_k$ has a “nice” meaning: perturbation of resonance mode or damping

\[ \begin{array}{c}
\text{frequency [Hz]} \\
\text{magnitude [dB]}
\end{array} \]

\[ \text{only } h_1 \text{ perturbed} \]

\[ \text{only } h_2 \text{ perturbed} \]

\[ \xi_1 \]

\[ \xi_2 \]
interpretation of perturbations

- $\xi_3$

- $\xi_4$
conclusions

- parametric uncertainty modeling:
  - additive uncertainty
  - small uncertainty bounds
  - physical intuition

- method based on experimental data

- reduced parameter models are obtained from linear regression analysis

- choice of nominal model + uncertainty structure

- realistic/data-based models for robust control design
end of presentation
In order to validate the model $\delta_k$ is taken at its extreme points so as to inspect the boundaries.

- exploring the boundaries $\delta=[-1,1]$
- too conservative at low frequencies
- too much freedom is given to the perturbations