Viterbi Detection for Partial Response Channels with Colored Noise

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Earlier research showed PR5 to be very useful for combating the effects of Thermal Asperities. A combiner with two independent processing channels (PR5 and EPR4) was proposed since the PR5 channel had poor AWGN performance.

Question: Is it possible to improve the AWGN performance of PR5 enough to dispense with the second channel and combining logic? Yes.
V. Dorfman and J.K. Wolf,
“A method for reducing the effects of Thermal Asperities”,
IEEE Journal on Selected Areas in Communications,
PR5 Channel \((1-D)^2(1+D)^2\)

Reduces the impact of Thermal Asperities in comparison with the base channel (EPR4). AWGN performance of PR5 is 2.5 dB worse than EPR4 because 1-D filter colors noise.

**NOTE:**
Can be implemented as interleave of two \((1-D)^2\) detectors.
Input \{0, 1\} → EPR4 Output

Colored Gaussian Noise, TA

→ PR5 Subchannel

→ TA Detector

→ EPR4 Subchannel

→ Combined Output To ECC

\{0, 1\}
EPR4 Channel Model, \( h_{EPR4} = (1-D)(1+D)^2 \)

Input \( \{0, 1\} \) → Write Head

Write Current \( \{-1, +1\} \) → Lorentzian Channel

Output \( \{0, 1\} \)

AWGN

Viterbi Detector (EPR4)

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PR5 Channel Model, \( h_{PR5} = (1-D)^2(1+D)^2 \)

Input \( \{0, 1\} \) → EPR4 CHANNEL

\( h_{EPR4}(D) = (1-D)(1+D)^2 \)

Output \( \{0, 1\} \)

AWGN
1. Equalizer colors the noise.
2. 1-D filter further colors the noise.
3. Viterbi detector is optimal for white noise, it computes $\min \sum_j (x_j - \hat{x}_j)^2$.
4. Viterbi detector is suboptimal for colored noise. It is required to compute $\min \sum_{i,j} q_{ij} (x_i - \hat{x}_i)(x_j - \hat{x}_j)$, which cannot be done by Viterbi algorithm.

$x_j$ – observed output including the noise at step $j$,
$\hat{x}_j$ - estimated noiseless output at step $j$,
Matrix $[q_{i,j}] = R^{-1}$, where $R = [r_{i,j}]$ is the covariance matrix of the noise at the equalizer output at step $i$ and step $j$. 
Difficult to compute \( \min \sum_{i,j} q_{ij} (x_i - \hat{x}_i)(x_j - \hat{x}_j) \)
for the whole sequence.

Instead, we attempt to compute

\[
\min \sum_i \{(x_i - \hat{x}_i)^2 + \sum_{k=1}^K C_k (x_i - \hat{x}_i)(x_{i-k} - \hat{x}_{i-k})\},
\]

where \( C_k \) are chosen experimentally to optimize
the bit error rate of the resulting detector
and \( K \) is a small number.
Use a trellis where each state represents K past states. Here K=2 is chosen. This increases the number of states 4 times, hence the term “Quad trellis”. In the new trellis each transition has information on K+1 previous transitions vs. only one for the original trellis.

Dashed and solid lines represent the “from” and “to” states in the new trellis.
EPR4 and PR5 - Standard and Quad Trellises, $D = PW50/T = 3.00$
SNR Required to Achieve 10^-5 BER vs. $D=PW50/T$
QUAD EPR4 and PR5 Improvements

SNR (dB) vs. D=PW50/T

- **QUAD PR5**
- **QUAD EPR4**
Use of the quad trellis for PR5 detection allows us to omit the EPR4 path and combining logic. The resulting AWGN performance is virtually the same as for the EPR4. However, the PR5 detector combats TA’s!!

Use of the quad trellis for EPR4 detection gains up to 1.5 dB.

No additional coding is required, the gains are achieved only by expanding the trellis.
TA PERFORMANCE OF EPR4, COMBINED, AND QUAD PR5 FOR D = PW50/T = 3.00

1.00E+02
1.00E+01
1.00E+00
1.00E-01
1.00E-02

Average bit errors per 1000 bit block with TA

TA Amplitude (x 0-to-peak magnetic readback signal)

- EPR4
- QUAD PR5
- PR5
Combating TA’s - Combining Approach

- **Viterbi Decoder for EPR4**
  - System Polynomial: \( h_{EPR4}(D) = (1-D)(1+D)^2 \)

- **Viterbi Decoder for EPR4**
  - System Polynomial: \( h_{EPR4}(D) = (1-D)^2 \)

- **Viterbi Decoder, 4 States**
  - System Polynomial: \( h_{PR5 interleaved}(D) = (1-D)^2 \)

- **Combating TA’s - Combining Approach**
  - Input: \( \{0, 1\} \)
  - EPR4 OUTPUT
  - COLORED GAUSSIAN NOISE, TA
  - Viterbi Decoder for EPR4
  - Averaging FIR
  - F \geq T
  - F < T

PR5

Combined Output

\( F \geq T \)

\( F < T \)