Decoding 2-Dimensional ISI Channels

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Decoding 2-dimensional ISI Channels

Overview

- Characterizing 2-D ISI Channels
- Optimal Decoding; 1-D vs. 2-D
- Iterative Decoding
- Decoding Bounds
- Simulation Results
- Summary & Future work
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2-Dimensional Channels

- \( b(t) = \int h(\tau)a(t - \tau).d\tau + n(t) \)
- and \( R(\tau) = E[n(t)n(t - \tau)] \)

- \( b(x,y) = \int \int h(p,q)a(x-p,y-q).dpdq + n(x,y) \)
- \( n(x,y), R(p,q) = E[n(x,y)n(x-p,x-q)] \)
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A simple 2-D Channel

\[ h = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \]

\[ h = \begin{bmatrix} 1 & .5 \\ .5 & 0 \end{bmatrix} \]
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Optimal Decoding 1-D

- Optimal decoding (MAP/ML) of 1-D finite-memory ISI of block length $N \sim O(2^\nu)$ complexity per bit

$$x_1, x_2, \ldots, x_j, x_{j+1}, \ldots, x_{j+\nu}, x_{j+\nu+1}, \ldots x_N$$

- $2^\nu$ states

- $x_1, x_2, \ldots, x_j$ independent of $x_{j+\nu+1}, \ldots x_N$ conditioned on a particular $s$

- Constant per-bit complexity a consequence of Markov property of finite state 1-D channel
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Optimal Decoding 2-D

- Boundary region grows with the number of bits decoded

- Complexity per bit $\sim O(2^\nu \sqrt{N})$

- Complexity $\rightarrow \infty$ as $N \rightarrow \infty$
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Iterative Decoding of 2-D ISI Channel

- Use a 1-D APP decoder (BCJR Algorithm) as a constituent block in an iterative decoder

- Unaccounted for ISI get treated as noise
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Decision Feedback Equalisation

- Straight-forward extension to higher dimensions

- No additional complexity for 2-D decoding $\sim O(n)$

- Sub-optimal
  - Only good for asymmetrical responses
  - Decision based on bit energy $1$ rather than $1 + \alpha^2$
  - Error propagation

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DFE-MAP Hybrid

- Perform MAP decoding on columns

- Make hard decision on previous columns and subtract out ISI (1-D DFE)

- Iterate as with MAP alone.
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Simple Decoding Bounds

2-D Matched Filter (MF) Bound

- Assume the whole energy of the response is concentrated in one bit-cell

![Graph showing 2-D matched filter bound](image)

1-D Matched Filter (MF) Bound

- Assume the energy in one dimension is concentrated in one bit-cell

![Graph showing 1-D matched filter bound](image)
Iterative 2-D Decoding $\alpha=0.5$

- MF Bound
- 1-D MF Bound
- ML
- dfe
- BCJR 1-D
- T-BCJR (5 Iter)
- T-BCJR (10 Iter)
- BCJR-DFE (2 Iter)
- BCJR-DFE (6 Iter)
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Iterative 2–D Decoding $\alpha=0.75$

![Graph showing BER vs. $E_b/N_0$ for various decoding methods including MF Bound, 1-D MF Bound, ML, dfe, BCJR 1-D, T–BCJR (5 Iter), T–BCJR (10 Iter), BCJR–DFE (2 Iter), BCJR–DFE (6 Iter).]
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SNR required to achieve 0.0001 BER

- MF Bound
- 1-D MF Bound
- ML
- dfe
- T-BCJR (5 Iter)
- T-BCJR (10 Iter)
- BCJR-DFE (2 Iter)
- BCJR-DFE (6 Iter)

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Summary and Future Work

- Optimal Decoding of 2-D ISI channels not realistic for large grids
- Iterative MAP on rows and columns does not perform well (cycles in graph)
- DFE gives good performance-complexity tradeoff for asymmetrical response
- DFE-MAP combination outperforms DFE or MAP alone
- Use MAP with higher order alphabet (multi-strip decoding)
- Apply post-processing methods
- Capacity Analysis
- Simulate more realistic channels